Competency Focused Practice Questions

Mathematics (Volume 1) | Grade 12

Co-created by

CBSE Centre for Excellence in Assessment

and

Educational Initiatives

Preface

Assessments are an important tool that help gauge learning. They provide valuable feedback about the effectiveness of instructional methods; about what students have actually understood and also provide actionable insights. The National Education Policy, 2020 has outlined the importance of competency-based assessments in classrooms as a means to reform curriculum and pedagogical methodologies. The policy emphasizes on the development of higher order skills such as analysis, critical thinking and problem solving through classroom instructions and aligned assessments.

Central Board of Secondary Education (CBSE) has been collaborating with Educational Initiatives (Ei) in the area of assessment. Through resources like the Essential Concepts document and A- Question-A-Day (AQAD), high quality questions and concepts critical to learning have been shared with schools and teachers.

Continuing with the vision to ensure that every student is learning with understanding, Question Booklets have been created for subjects for Grade 10th and 12th. These booklets contain competency-based items, designed specifically to test conceptual understanding and application of concepts.

Process of creating competency-based items

All items in these booklets are aligned to the NCERT curriculum and have been created keeping in mind the learning outcomes that are important for students to understand and master. Items are a mix of Free Response Questions (FRQs) and Multiple-Choice Questions (MCQs). In case of MCQs, the options (correct answer and distractors) are specifically created to test for understanding and capturing specific errors/misconceptions that students may harbour. Each incorrect option can thereby inform teachers on specific gaps that may exist in student learning. In case of subjective questions, each question also has a detailed scoring rubric to guide evaluation of students' responses.

Each item has been reviewed by experts, to check for appropriateness of the item, validity of the item, conceptual correctness, language accuracy and other nuances.

How can these item booklets be used?

There are 267 questions in this booklet.

The purpose of these item booklets is to provide samples of high-quality competency-based items to teachers. The items can be used to–

- get an understanding of what good competency-based questions could look like
- give exposure to students to competency-based items
- assist in classroom teaching and learning
- get inspiration to create more such competency-based items

Students can also use this document to understand different kinds of questions and practice specific concepts and competencies. There will be further additions in the future to provide competency focused questions on all chapters.

The item booklets are aligned with the 2022-23 curriculum. However, a few questions from topic which got rationalized in 2023-24 syllabus are also there in the booklet which may be used as a reference for teachers and students.

Please write back to us to give your feedback.

Team CBSE

Table of Contents

Chapter - 1
Relations and Functions

Q: 1 **Given below is a relation R from the set X = { x, y, z } to itself.**

R = {(x, x), (x, y), (y, x), (y, z), (x, z)}

Which of the following is true about the relation R?

1. R is reflexive and transitive but not symmetric.
2. R is symmetric and transitive but not reflexive.
3. R is transitive but neither reflexive nor symmetric.
4. R is not reflexive, not symmetric and not transitive.

Q: 2 **A and B are two sets with m elements and n elements respectively ($m < n$).**

How many onto functions can be defined from set A to B?

1. 0
2. $m\,!$
3. $n\,!$
4. n^{m}

Q: 3 **Three students Aabha, Bhakti and Chirag were asked to define a function, f, from set X = {1, 3, 5, 7, 9} to set Y = {2, 4, 6, 8}. Their responses are shown below:**

Aabha: f = {(1, 2), (1, 4), (1, 6), (1, 8)}

Bhakti: f = {(1, 2), (3, 4), (5, 6), (7, 8)}

Chirag: f = {(1, 4), (3, 4), (5, 4), (7, 4), (9, 4)}
Who defined a function correctly?

1. only Chirag
2. only Aabha and Bhakti
3. only Bhakti and Chirag
4. only Chirag and Aabha

Q: 4 **Which of the following is an equivalence relation on the set P = {1, 4, 9}?**

1. R_1 = {(1, 1), (4, 4), (9, 9)}
2. R_2 = {(1, 1), (4, 4), (1, 4), (4, 1)}
3. R_3 = {(4, 4), (9, 9), (1, 1), (9, 1), (1, 9), (1, 4), (4, 1)}
4. R_4 = {(1, 4), (4, 4), (9, 4), (4, 1), (1, 1), (9, 9), (9, 1)}

Q: 5 **The power set of a set A = { a, b } is the set of all subsets of A. These subsets are given by:**

P(A) = {Φ, { a }, { b }, { a, b }}

A relation R is defined on P(A) as R = {(r, s) : $r \subseteq s$ }

Which of the following is the correct representation of R in its roster form?

1. {Φ, { a }, { b }, { a, b }}
2. {(Φ, { a }), (Φ, { b }), (Φ, { a, b })}
3. {(Φ, { a }), (Φ, { b }), (Φ, { a, b }), ({ a }, { a, b }), ({ b }, { a, b })}
4. {(Φ, { a }), (Φ, { b }), (Φ, { a, b }), ({ a }, { a, b }), ({ b }, { a, b }), ({ a, b }, { a, b })}

Q: 6 ***f*** **: X -> X is a function on the finite set X.**

Given below are two statements based on the above context - one labelled Assertion (A) and the other labelled Reason (R). Read the statements carefully and choose the option that correctly describes statements (A) and (R).

Assertion (A) **: If *f* is onto, then *f* is one-one and if *f* is one-one, then *f* is onto.**

Reason (R) **: Every one-one function is always onto and every onto function is always one-one.**

1. Both (A) and (R) are true and (R) is the correct explanation for (A).
2. Both (A) and (R) are true but (R) is not the correct explanation for (A).
3. (A) is true but (R) is false.
4. Both (A) and (R) are false.

Q: 7 **The set of non-negative integers, denoted by Z^*, is the set containing positive integers along with zero.**

On the set Z^*, a function *f* : Z^* -> Z^* is defined by:

$$f(n) = \begin{cases} \frac{n+1}{2}, \text{ if } n \text{ is an odd integer} \\ 0 \text{ , if } n \text{ is an even integer} \end{cases}$$

Which of the following is true about *f* ?

1. *f* is one-one but not onto
2. *f* is onto but not one-one
3. *f* is both one-one and onto
4. *f* is neither one-one nor onto

Q: 8 **Consider an operation * defined on the set { *a* , *b* , *c* } given by the following operation table.**

*	***a***	***b***	***c***
a	*a*	*a*	*a*
b	*a*	*a*	*a*
c	*a*	*a*	*a*

Which of the following is true about the operation *?

1. * is not a binary operation
2. * is a binary operation that is commutative but not associative
3. * is a binary operation that is associative but not commutative
4. * is a binary operation that is both commutative and associative

Q: 9 **If $f(x) = x^3 + 1$ and $f(g(x)) = x$, then which of the following is $g(1)$?**

1. 0
2. 1
3. 2
4. (cannot be determined without knowing what is $g(x)$)

Q: 10 **A relation R on set G = {All the students in a certain mathematics class} is defined as, R = {(x, y): x and y have the same mathematics teacher}.**

Which of the following is true about R?

1. R is reflexive and transitive but not symmetric.
2. R is transitive and symmetric but not reflexive.
3. R is reflexive and symmetric but not transitive.
4. R is an equivalence relation.

Q: 11 **State whether the following statement is true or false. Justify your answer.** **[1]**

"The sine function is bijective in nature when the domain is set from 0 to 4π."

Q: 12 **[1]**

$$f(x) = \frac{2\tan x}{1-\tan^2 x}$$

Find the range of $f(x)$ for $x \in R$. Show your steps.

Q: 13 **State whether the following statement is true or false. Justify your answer.** **[1]**

"A function $f(x) = \ln x$ is invertible for all values of x."

Q: 14 **[1]**

A = {1, 3, 5, 7, ...}
B = {2, 4, 6, 8, ...}

Define a function from A to B that is neither one-one nor onto.

Q: 15 **X and Y are two sets with their number of elements being k and l respectively ($k < l$).** **[1]**

Find the number of onto functions that can be defined from set X to Y. Explain your answer.

Q: 16 **Is * defined on Q by $m * n = m^n$ a binary operation? Justify your answer.** **[1]**

(Note: Q is the set of rational numbers.)

Q: 17 **Find the domain of (*f* o *g*) if** **[2]**

$$f(x) = \frac{3}{x+1}$$

and

$$g(x) = \frac{x}{3x-2}$$

Show your steps and give reasons.

Q: 18 **Shreyas and Simran are playing a game in which they are trying to guess relations on set A = {-2, -3}. Simran tells Shreyas that she is thinking of an equivalence relation.** **[2]**

Shreyas guesses the relation as R = {(-2, -3), (-3, -2), (-3, -3)} .

Could Shreyas be correct? Justify your answer.

Q: 19 **Let *f* : A → B and *g* : B → C be two functions.** **[2]**

i) If *f* is not one-one, can *g* o *f* be one-one?
ii) If *f* is not onto, can *g* o *f* be onto?

Justify your answer.

Q: 20 **f: R -> R defined by f(*x*) = $\frac{3x}{8-5x}$ is not a function.** **[2]**

i) Why is f not a function?
ii) Based on your explanation in part i), what changes can be made so that f(*x*) = $\frac{3x}{8-5x}$ becomes a function?

Q: 21 **The operations table for a binary operation * is shown below.** **[2]**

*	3	5	7
3	3	3	3
5	3	5	5
7	3	5	7

i) Define the above binary operation *.
ii) Is the operation commutative?
iii) Is the operation associative?

Justify your answer.

Q: 22 **State whether the following statement is true or false. If true, give a reason. If false, give an example.** **[2]**

If $f(x)$ and $g(x)$ are two functions, then their composition is commutative. In other words, $f(g(x)) = g(f(x))$.

Q: 23 **A teacher wrote $f(x) = x^9$ on the board and asked her student to examine whether the following equation is true or false.** **[2]**

$$[f(x)]^{\frac{1}{3}} = [f^{-1}(x)]^3$$

Raghu said, "It is true".

Is he correct? Justify your answer.

Q: 24 **f and g are real functions such that f is bijective and $g(x) = 3x + 4$, for all $x \in R$.** **[3]**

Is (gof) invertible? Justify your answer.

Q: 25 **X = {2, 6, 12, 20, ...}** **[3]**

Y = {2, 3, 4, 5, ...}

i) Define a one-one function from set X to set Y.
ii) Show that the function you defined is one-one.

Q: 26 **A relation R in set G = {All the countries in the world} is defined as R = {(x, y): x and y share a common boundary}.** **[3]**

Determine whether R is reflexive, symmetric and transitive. Hence, conclude if R is an equivalence relation. Show your work.

Q: 27 **Graph of a certain function f: R -> R is a straight line parallel to x -axis, where R is the set of real numbers.** **[3]**

i) Is the function one-one?
ii) Is the function onto?

Justify your answer.

Read the information given below and answer the questions that follow.

A confectionery shop is a place where sweets and chocolates are sold. The table below gives information on four varieties of chocolates sold there.

Chocolate name	Cost per piece (in Rs)
Daily Milk (D)	5
25-Star (S)	10
Crunch (C)	20
Ket-Kat (K)	50

Let A = {D, S, C, K} be the set containing the chocolates and B = {5, 10, 20, 50} be the set containing their costs.

A relation R is defined on set A as R = {(x, y) : cost of $x \leq$ cost of y }.

Q: 28 **Express the relation R in roster form.** **[1]**

Q: 29 **Is R a reflexive relation? Justify your answer.** **[1]**

Q: 30 **Is R a symmetric relation? Justify your answer.** **[1]**

Q: 31 **Is R a transitive relation? Justify your answer.** **[1]**

Q: 32 **Define a function from set A to set B.** **[1]**

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	4
2	1
3	1
4	1
5	4
6	3
7	2
8	4
9	1
10	4

Q.No	Teacher should award marks if students have done the following:	Marks
11	**Writes False(F).**	**0.5**
	Writes that the sine function is onto but not one-one (gives an example such as sin($\frac{\pi}{2}$) = sin($\frac{5\pi}{2}$) = 1), therefore it is not bijective in nature.	**0.5**
12	**Rewrites *f* (*x*) as tan(2 *x*).**	**0.5**
	Writes that the range of tan(2 *x*) is R(all real numbers).	**0.5**
13	**Writes True(T).**	**0.5**
	Writes that the logarithmic function is both onto and one-one. Hence, its inverse exists. **(Award full marks for any other logical explanation.)**	**0.5**
14	**Defines a function from A to B that is neither one-one nor onto. For example,** **f: A -> B defined by f(*x*) = 4 for all *x* ∈ A.**	**1**
15	**Writes that the number of onto functions from set X to Y is zero.**	**0.5**
	Reasons that, since set Y contains more elements than set X, at least one element of Y will always remain unmapped.	**0.5**
16	**Writes no.**	**0.5**
	writes a pair of rational numbers *m* and *n* such that m^n is not rational. **For example, 2 * $\frac{1}{2}$ = $2^{\frac{1}{2}}$ = √2 ∉ Q.**	**0.5**
17	**Finds the composite function (*f* o *g*)(*x*) as:** $\frac{3(3x-2)}{2(2x-1)}$	**0.5**

Q.No	Teacher should award marks if students have done the following:	Marks
	Writes that $\frac{1}{2} \notin$ domain of (f o g) as (f o g)(x) is not defined at $x = \frac{1}{2}$.	0.5
	Writes that the domain of (f o g) will not contain $\frac{2}{3}$ as the domain of (f o g) is a subset of the domain of g, and $\frac{2}{3} \notin$ domain of g.	0.5
	Concludes that the domain of (f o g) is the set of all real numbers except $\frac{1}{2}$ and $\frac{2}{3}$.	0.5
18	Writes that Shreyas is not correct.	0.5
	Writes that Shreyas' relation is not reflexive as (-2, -2) is not a part of it.	1
	Writes that, since the relation is not reflexive, it cannot be an equivalence relation.	0.5
19	i) Writes that g o f cannot be one-one, since g o f is one-one implies f is one-one.	1
	ii) Writes that g o f can be onto, since g o f is onto implies g is onto and there is no restriction on f to be one-one or onto.	1
20	i) Writes that f($\frac{8}{5}$) = $\frac{24}{0} \notin$ R. Thus, f is not a function as it is not well defined.	1
	ii) Writes that f can be made a function by removing the element $\frac{8}{5}$ from the domain R.	1
21	i) Defines the given binary operation as $a * b$ = min{ a, b }.	0.5
	ii) Writes that the function is commutative as min{ a, b} = min{ b, a }. or Writes that the function is commutative since first row is identical to first column in the operations table.	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	iii) Writes that, $a * (b * c) = a * \min\{b, c\} = \min\{a, b, c\}$. Similarly, $(a * b) * c = \min\{a, b\} * c = \min\{a, b, c\}$. **Therefore, $a * (b * c) = (a * b) * c$ and the binary operation is associative.**	**1**
22	**Writes false.**	**0.5**
	Gives an example: **Let $f(x) = x^2$ and, $g(x) = x + 2$.** **Then, $f(g(x)) = (x + 2)^2$ and, $g(f(x)) = x^2 + 2$.** **=> $f(g(x)) \neq g(f(x))$**	**1.5**
23	**Writes that Raghu is wrong.**	**0.5**
	Finds $f^{-1}(x)$ as $x^{\frac{1}{9}}$.	**0.5**
	Finds RHS as $[f^{-1}(x)]^3 = x^{\frac{1}{3}}$.	**0.5**
	Finds LHS as $[f(x)]^{\frac{1}{3}} = x^3$ and compares it with the RHS. **Concludes that $[f(x)]^{\frac{1}{3}} \neq [f^{-1}(x)]^3$.**	**0.5**
24	**Writes that $g(x)$ is one-one as:** **$g(x_1) = g(x_2)$** **$\Rightarrow 3x_1 + 4 = 3x_2 + 4$** **$\Rightarrow x_1 = x_2$.**	**0.5**
	Writes that for any real value of y in R, there exists $\frac{y-4}{3}$ in R such that: **$g\left(\frac{y-4}{3}\right)$** **$= 3 \times \frac{y-4}{3} + 4$** **$= y - 4 + 4$** **$= y$** **Thus, $g(x)$ is onto.** **Concludes that $g(x)$ is bijective therefore invertible.**	**1**

Q.No	Teacher should award marks if students have done the following:	Marks
	Writes that $f(x)$ is bijective and therefore invertible.	0.5
	Uses the theorem that if f and g are invertible, then (gof) is also invertible to conclude that (gof) is invertible.	1
25	i) Rewrites set X as $\{1 \times 2, 2 \times 3, 3 \times 4, 4 \times 5, ..., n(n+1), ...\}$.	1
	Defines a function f: X -> Y as: $f(n(n+1)) = n + 1$	1
	ii) Shows that the above function is one-one as: $f(n(n+1)) = f(m(m+1))$ $\Rightarrow n + 1 = m + 1$ $\Rightarrow n = m$	1
26	Writes that every country shares its boundary with itself. That is, $(x,x) \in R$, for each element $x \in G$. Hence, R is reflexive.	0.5
	Writes that, whenever x shares a boundary with y, y also shares a boundary with x. That is, $(x,y) \in R \Rightarrow (y,x) \in R$. Hence, R is symmetric.	1
	Writes that, if x shares a boundary with y and y shares a boundary with z, then x need not share a boundary with z. That is, $(x,y) \in R$, $(y,z) \in R$ need not imply $(x,z) \in R$. Hence, R is not transitive.	1
	From the above steps, concludes that R is not an equivalence relation.	0.5
27	i) Writes that the function must be of the form $f(x) = k$, where 'k' is a real number.	1
	Writes that f is not one-one. Justifies by giving an example as follows: $f(1) = k = f(2)$, but $1 \neq 2$.	1

Q.No	Teacher should award marks if students have done the following:	Marks
	ii) Writes that f is not onto. Justifies by giving an example as follows: Consider β ($\neq k$) $\in$ R (codomain), there is no element $x \in$ R (domain) such that f(x) = β.	1
28	Expresses the relation R in roster form as R = {(D, S), (D, C), (D, K), (S, C), (S, K), (C, K)} .	1
29	Writes yes.	0.5
	Justifies the answer. For example, the cost of every chocolate is equal to its own cost i.e. (x, x) $\in$ R, for every $x \in$ A.	0.5
30	Writes no.	0.5
	Justifies the answer. For example, (D, S) $\in$ R but (S, D) $\notin$ R. Hence, R is not symmetric.	0.5
31	Writes yes.	0.5
	Justifies the answer. For example: Let (x, y) and (y, z) $\in$ R => cost of $x \leq$ cost of y and cost of $y \leq$ cost of z Uses the above set of inequalities to show that cost of $x \leq$ cost of z. Hence, concludes that (x, z) $\in$ R.	0.5
32	Defines a function from set A to set B. For example: f : A -> B, defined by, f (x) = cost of x. (Award full marks if any function is written correctly in set-builder form or in roster form.)	1

Chapter - 2
Inverse Trigonometric Functions

Q: 1 **What is the domain of the function $y = \sec^{-1} x + \sin^{-1} x$?**

1. -1 and 1
2. [-1, 1]
3. (-∞, -1] ∪ [1, ∞)
4. Φ

Q: 2

Given: $\tan^{-1}\left(\sqrt{1+\tan^2\theta} \times \sin(\pi - \theta)\right) = \dfrac{1-a^2}{(1+a)^2}$

where $a \in R$ and $a \neq -1$

Which of the following gives the value of θ in terms of a?

$\tan\left(\dfrac{a-1}{a+1}\right)$ — Expression 1

$\left(\dfrac{1+a}{1-a}\right)$ — Expression 2

$\dfrac{1-a^2}{(1+a)^2}$ — Expression 3

$\left(\dfrac{a-1}{1+a}\right)$ — Expression 4

1. Expression 1
2. Expression 2
3. Expression 3
4. Expression 4

Q: 3 **Which of the following is the domain of the function given below?**

$y = \cos^{-1}\left(\frac{1}{x-3}\right)$

1. [-1, 1]
2. [2, 4]
3. (-∞, 2] ∪ [4, ∞)
4. (-∞, -1] ∪ [1, ∞)

Q: 4 **The domain of $f(x) = \sin^{-1} x - \cos^{-1} x$ is [-1, 1] while its range is $\left[\frac{-3\pi}{2}, \frac{\pi}{2}\right]$.**

If $g(x)$ is the inverse of $f(x)$, which of the following is true about the domain of the function $g(x)$?

1. It is [-1, 1].
2. It is $\left[\frac{-3\pi}{2}, \frac{\pi}{2}\right]$.
3. It is independent of the domain and range of f(x).
4. (cannot be said without knowing $g(x)$)

Q: 5 **If $\sec^{-1}(-x) = \frac{\pi}{8}$, which of the following could be the value of $\sec^{-1}(x)$?**

1. $\left(-\frac{\pi}{8}\right)$
2. $\frac{7\pi}{8}$
3. $\frac{9\pi}{8}$
4. (cannot be determined without knowing the value of x)

Q: 6 **Akash says that, since the domain of the sine function is (-∞, ∞), $\sin^{-1} x$ is well defined in the domain (-∞, ∞).** **[1]**

Is Akash right or wrong? Justify your answer.

Q: 7 **Simplify:** **[1]**

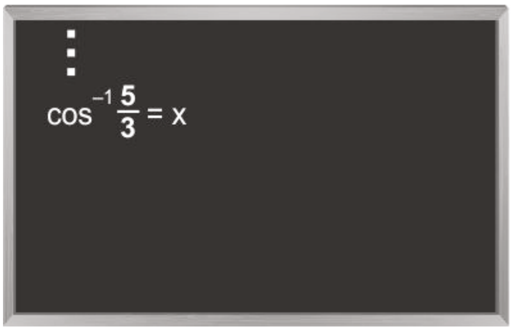
$\cos\left(\frac{\pi}{2} + \sin^{-1}\frac{1}{\sqrt{3}}\right)$

Show your work.

Q: 8 $\operatorname{cosec}\frac{\pi}{6} = \operatorname{cosec}\frac{5\pi}{6} = 2$, **but** $\frac{\pi}{6} \neq \frac{5\pi}{6}$. **[1]**

Since the cosecant function is not one-one, how can it be made invertible? Give a reason for your answer.

Q: 9 **While solving an inverse trigonometry problem on the blackboard, Satish wrote the following as part of his solution:** **[1]**



His teacher stopped him and said that he must have made a mistake in the solution.

How did the teacher recognise that Satish had made a mistake? Justify your answer.

Q: 10 **What would be the value of** $\cos^{-1}\left(\frac{24}{25}\right) + \tan^{-1}\left(\frac{24}{7}\right)$**?** **[2]**

Q: 11 **Prove that:** **[2]**

$$\sin^{-1}\left[\frac{2^{x+1}}{1+4^{x}}\right] = 2\tan^{-1}(2^{x}), \text{ where } x \leq 0$$

Q: 12 **Chirag asked his students to find the value of:** **[2]**

$\tan^{-1}(-x) - \tan^{-1}\left(\frac{1}{x}\right)$

Rahul said that the value of the above expression can be found ONLY if *x* is known.

Is Rahul correct? Justify your answer.

Q: 13 **Considering the principal value branch, prove that the property below is true ONLY for $xy > (-1)$.** **[3]**

$$\tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy}$$

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	1
2	3
3	3
4	2
5	2

Q.No	Teacher should award marks if students have done the following:	Marks
6	**Writes that Akash is wrong.**	0.5
	Writes that $\sin^{-1} x$ cannot be defined in the domain $(-\infty, \infty)$ as $\sin x$ is not one-one in that domain. **(Award full marks for any other valid reason.)**	0.5
7	**Simplifies the above expression as:** $\cos(\frac{\pi}{2} + \sin^{-1}\frac{1}{\sqrt{3}}) = -\sin(\sin^{-1}\frac{1}{\sqrt{3}})$	0.5
	Simplifies the above expression as $(-\frac{1}{\sqrt{3}})$.	0.5
8	**Writes that the cosecant function can be made invertible when its domain is restricted to $[n\pi - \frac{\pi}{2}, n\pi + \frac{\pi}{2}] - \{0\}$, where n is an integer.**	1
9	**Writes that the $\cos^{-1}$ function is not defined for $\frac{5}{3}$ as it is outside the domain of the inverse cosine function. Therefore, the teacher stopped Satish here.**	1
10	**Assumes $\cos^{-1}(\frac{24}{25})$ as x and writes that $\cos x = \frac{24}{25}$.**	0.5
	Uses the above step and writes: $\cot x = \frac{24}{7}$ $\Rightarrow x = \cot^{-1}(\frac{24}{7})$	1
	Uses the property, $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$, for $x \in R$, and evaluates the given expression as: $\cot^{-1}(\frac{24}{7}) + \tan^{-1}(\frac{24}{7}) = \frac{\pi}{2}$ **(Award full marks if the problem is solved correctly using any other method.)**	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
11	**Rewrites the LHS of the given equation as:** $$\sin^{-1}\left[\frac{2^{x+1}}{1+4^{x}}\right] = \sin^{-1}\left[\frac{2.\,2^{x}}{1+(2^{x})^{2}}\right]$$	1
	Uses the property of inverse trigonometric functions and writes: $$\sin^{-1}\left[\frac{2.\,2^{x}}{1+(2^{x})^{2}}\right] = 2\tan^{-1}(2^{x})$$	1
12	**Writes that Rahul is incorrect.**	0.5
	Writes that $\tan^{-1}(-x) - \tan^{-1}\frac{1}{x}$ **can be written as** $-(\tan^{-1}x + \cot^{-1}x)$.	1
	Finds the value of the given expression as $(-\frac{\pi}{2})$ **as** $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$.	0.5
13	**Assumes** $xy < (-1)$, $x = \tan\theta$ **and** $y = \tan\Phi$.	0.5
	Rewrites the above inequality as $\tan\theta < \tan(\Phi - \frac{\pi}{2})$.	0.5
	Writes that, since tangent is an increasing function in the principal value branch, $\theta < (\Phi - \frac{\pi}{2})$.	0.5
	Uses steps 1 and 3 to write $\tan^{-1}x - \tan^{-1}y < -\frac{\pi}{2}$.	0.5
	Uses the above step to conclude that: **If** $xy < (-1)$, **the value of** $\tan^{-1}x - \tan^{-1}y \notin (-\frac{\pi}{2}, \frac{\pi}{2})$.	0.5
	Hence, proves that the given property is true only for $xy > (-1)$.	0.5

Chapter - 3
Matrices

Q: 1 **T is a matrix given by:**

$$T = \begin{bmatrix} 4 & 0 & 4 \\ 4 & 0 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

On performing which of the following individual operations, will the matrix T remain the same?

i) $C_1 \leftrightarrow C_3$

ii) $R_2 \rightarrow R_2 + 99R_3$

iii) $C_2 \rightarrow (-1)C_2$

1 only (i) and (ii)
2 only (ii) and (iii)
3 only (i) and (iii)
4 all - (i), (ii) and (iii)

Q: 2 **A teacher gave the following problem to his students.**

$$\begin{bmatrix} 3 & 7 & 2 \\ -1 & 4 & 0 \end{bmatrix} \times \begin{bmatrix} 5 & 4 & -2 \\ 6 & 3 & 8 \end{bmatrix}$$

Three students' solutions are shown below.

$\begin{bmatrix} 15 & 28 & -4 \\ -6 & 12 & 0 \end{bmatrix}$ Ram's solution

$\begin{bmatrix} 39 & 55 \\ 11 & 6 \end{bmatrix}$ Aviraj's solution

$\begin{bmatrix} 57 & 33 & 50 \\ 19 & 8 & 34 \end{bmatrix}$ Shyama's solution

Who gave the correct answer?

1 only Ram
2 only Aviraj
3 only Shyama
4 none of them

Q: 3

A and B are two matrices such that the transpose of (A + B) is $\begin{bmatrix} 2 & -1 \\ 7 & 6 \end{bmatrix}$

If $B = \begin{bmatrix} 3 & -2 \\ 6 & 0 \end{bmatrix}$, which of the following is A?

1 $\begin{bmatrix} -1 & 1 \\ 1 & 6 \end{bmatrix}$
2 $\begin{bmatrix} -1 & 9 \\ -7 & 6 \end{bmatrix}$
3 $\begin{bmatrix} 1 & -9 \\ 7 & -6 \end{bmatrix}$
4 $\begin{bmatrix} -4 & 4 \\ 0 & 7 \end{bmatrix}$

Q: 4 **In an online advertisement (ad) campaign, there are two options for publicity: picture ads and video ads.**

The cost per ad (in Rs) is given by the matrix:

$$A = \begin{array}{cc} \text{Cost per ad} & \\ \begin{bmatrix} 0.2 \\ 0.5 \end{bmatrix} & \begin{array}{l} \text{Picture} \\ \text{Video} \end{array} \end{array}$$

The number of ads run by two companies X and Y is given by the matrix:

$$B = \begin{array}{cc} \begin{array}{cc} \text{Picture} & \text{Video} \end{array} & \\ \begin{bmatrix} 10000 & 15000 \\ 8000 & 20000 \end{bmatrix} & \begin{array}{l} X \\ Y \end{array} \end{array}$$

To find the total cost of ads for the two companies, Nahush performs the matrix operation A × B while Divyesh performs the matrix operation B × A.

Who is correct?

1. Only Nahush
2. Only Divyesh
3. Both Nahush and Divyesh
4. Neither Nahush nor Divyesh

Q: 5

$A = \begin{bmatrix} 3 & 4 \\ -5 & 2 \end{bmatrix}$ and B is the inverse of A.

Which of the following is AB?

1. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
2. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
3. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
4. $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Q: 6 **A and B are two matrices such that both products, AB and BA, exist.** **[1]**

Write a condition on the order of the matrices A and B for the above statement to be true.

Q: 7 **Identify if the statement below is true or false. If true, give a reason. If false, give a counter-example.** **[1]**

"If A is a non-zero matrix such that A × K is a zero matrix, then K is definitely a zero matrix."

Q: 8 **The matrix obtained after applying the column operation [$C_1 \rightarrow C_1 + (-3)C_3$] on matrix A is shown below.** **[1]**

$$\begin{bmatrix} 5 & 5 & -2 \\ 1 & -6 & 1 \\ -6 & -4 & 3 \end{bmatrix}$$

Find matrix A. Show your work.

Q: 9 **A is a 4 × 3 matrix, which when multiplied by matrix B, results in a 4 × 2 matrix, C.** **[1]**

How many rows and columns does matrix B have? Justify your answer.

Q: 10 **[2]**

$$A = \begin{bmatrix} 2 & -3 \\ 1 & 0 \end{bmatrix}$$

If $p\,A^2 + q\,A + r\,I = O$, where O is the zero matrix and p, q and r are integers, find the value of ($\frac{-q}{p}$). Show your steps.

Q: 11 **If B is a symmetric matrix, prove that BB' is a symmetric matrix.** **[2]**

Q: 12 **D is a matrix of order 3 which is both symmetric and skew symmetric.** **[2]**

Find D. Show your work.

Q: 13 **A, B and C are three matrices that are compatible for multiplication. Under what condition(s) will the following statement be true?** **[2]**

If AB = AC, then B = C.

Show your working.

Q: 14 **D is a diagonal matrix and BD = I, where B is a matrix.** **[2]**

Is B a diagonal matrix? Justify your answer.

Q: 15 Two distributors of a chips company distribute two varieties of chips packets - Spicy Chips (SC) and Cheesy Chips (CC). **[3]**

At the beginning of a certain month, the number of chips packets available with the distributors is shown in matrix M. The number of chips packets distributed during that month by them is shown in matrix N.

$$M = \begin{array}{c} \begin{array}{cc} SC & CC \end{array} \\ \begin{bmatrix} 965 & 498 \\ 872 & 689 \end{bmatrix} \end{array} \begin{array}{l} \\ \longrightarrow \text{Distributor 1} \\ \longrightarrow \text{Distributor 2} \end{array}$$

$$N = \begin{array}{c} \begin{array}{cc} SC & CC \end{array} \\ \begin{bmatrix} 956 & 399 \\ 650 & 511 \end{bmatrix} \end{array} \begin{array}{l} \\ \longrightarrow \text{Distributor 1} \\ \longrightarrow \text{Distributor 2} \end{array}$$

i) Find the number of chips packets remaining with the distributors at the end of that month.

ii) A packet of Spicy Chips and Cheesy Chips costs Rs 10 and Rs 20 respectively. Find the total cost of the chips distributed by each distributor that month using matrix multiplication.

Show your work and give your answer in the matrix form.

Q: 16 **[3]**

$A = \begin{bmatrix} 1 & 1 \\ 0 & \beta \end{bmatrix}; \; B = \begin{bmatrix} 1 & 2 \\ 0 & 9 \end{bmatrix}$

For what value(s) of β, is $A^2 = B$? Show your work.

Q: 17 **[3]**

Given:

$(AB) \times C = \begin{bmatrix} -13 & 12 \\ 32 & 11 \end{bmatrix}$ and $A = \begin{bmatrix} 2 & -3 \\ 7 & 5 \end{bmatrix}$

Find BC. Show your work with valid reasons.

Q: 18 **[3]**

$A = \begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 3 & 4 \\ 4 & 2 \end{bmatrix}$

i) Find the relation between the elements of matrix B, such that AB = BA. Show your steps.
ii) Use the relations from part i) to justify if the matrix C satisfies AC = CA.

Q: 19 **A is a square matrix of order m.** **[3]**

If $(A^2 - A)$ is invertible, then is A invertible? Justify your answer.

Q: 20 **Achal wants to purchase 2 kg of sugar, 10 kg of wheat and 5 kg of rice. In a general store near his house, these groceries were priced at Rs 50, Rs 35 and Rs 40 per kg whereas in a supermarket, these groceries were priced at Rs 44, Rs 30 and Rs 38 per kg respectively. The cost of travelling to the supermarket is Rs 20.** **[3]**

Using matrix multiplication, find Achal's total savings if he buys the groceries from supermarket. Show your work.

Q: 21 **A company wanted to outsource the creation of its video and picture content for social media.** **[5]**

They were approached by two firms offering the following rates per post (in Rs).

$$\begin{array}{cc} \text{Video} & \text{Picture} \\ \end{array}$$
$$\begin{bmatrix} 30 & 10 \\ 25 & 15 \end{bmatrix} \begin{array}{l} \text{Firm 1} \\ \text{Firm 2} \end{array}$$

They gave the contract to both the firms for an equal number of posts. After 3 months, each firm created the following number of posts:

$$\begin{bmatrix} 70 \\ 100 \end{bmatrix} \begin{array}{l} \text{Video} \\ \text{Picture} \end{array}$$

Each firm's posts created the following number of sales for the company:

$$\begin{array}{cc} \text{Video} & \text{Picture} \\ \end{array}$$
$$\begin{bmatrix} 500 & 200 \\ 400 & 300 \end{bmatrix} \begin{array}{l} \text{Firm 1} \\ \text{Firm 2} \end{array}$$

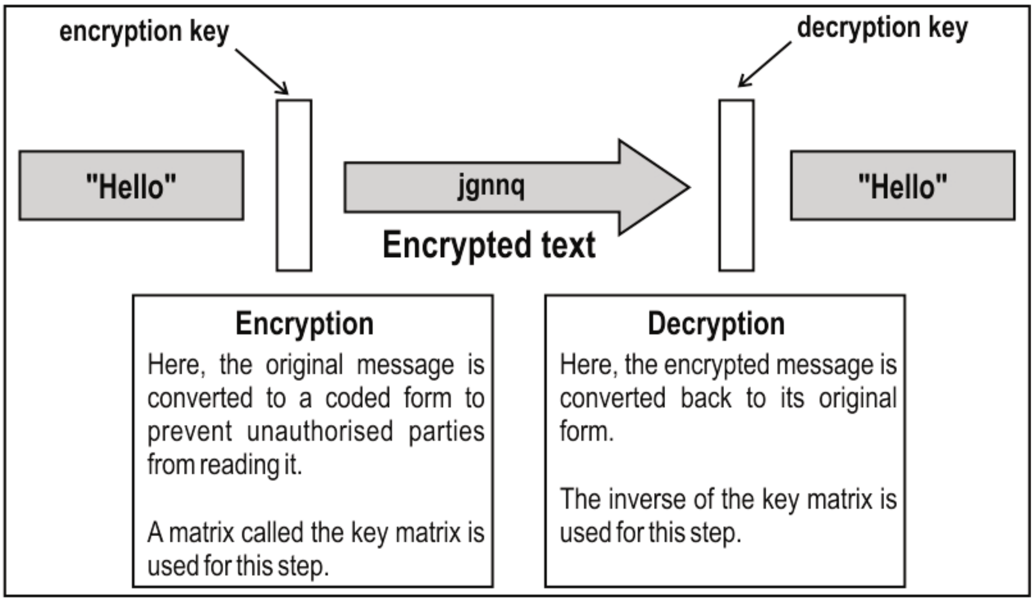
i) Which firm cost more to the company?

ii) If each sale was worth Rs 300, which firm was more profitable for the company?

Show your work.

Study the given information and answer the questions that follow.

One of the prominent applications of matrices is in cryptography. Cryptography is a type of secure communication where a message is transmitted from a sender to a receiver.



An encryption process with a key matrix, K = $\begin{bmatrix} 3 & 7 \\ 4 & 1 \end{bmatrix}$ is shown below.

Assume the coordinates of a location, P = (4, 18), is to be encrypted. Matrix multiplication is performed between the key matrix and the coordinates to obtain the encrypted form of P as:

$$K \times P = \begin{bmatrix} 3 & 7 \\ 4 & 1 \end{bmatrix} \times \begin{bmatrix} 4 \\ 18 \end{bmatrix} = \begin{bmatrix} 138 \\ 34 \end{bmatrix}$$

Decryption is carried out by using the inverse of the key matrix as:

K^{-1} × encrypted matrix = original matrix

Q: 22 **Using the same key matrix, K, find the encrypted form of Q = (19, -20).** **[1]**

Q: 23 **Find the matrix used for decryption corresponding to K. Show your work.** **[2]**

Q: 24 **If M = (160, 105) is the encrypted coordinates corresponding to the key matrix K, find the original coordinates of M.** **[1]**

Q: 25 **K_1 is the new key matrix obtained by performing the elementary row operation $R_2 \to R_2 + R_1$ on K, where R_1 and R_2 denote rows 1 and 2 respectively.** **[1]**

Find K_1.

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	4
2	4
3	2
4	2
5	2

Q.No	Teacher should award marks if students have done the following:	Marks
6	**Writes that the above statement will be true if the orders of matrices A and B are of the form $m \times n$ and $n \times m$ respectively, where m and n are positive integers.** **(Award 0.5 marks if the condition 'both the matrices A and B are square matrices of the same order' is written.)** **OR** **(Award 0.5 marks if particular orders are written instead. For example, 2 × 3 and 3 × 2.)**	1
7	**Writes False(F).**	0.5
	Gives a counterexample. For example: $\begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix} \times \begin{bmatrix} 5 & -3 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ where $A = \begin{bmatrix} 0 & 2 \\ 0 & 7 \end{bmatrix}$ & $K = \begin{bmatrix} 5 & -3 \\ 0 & 0 \end{bmatrix}$	0.5
8	**Applies the reverse operation $[C_1 \rightarrow C_1 + 3C_3]$ on the given matrix to find matrix A as:** $\begin{bmatrix} -1 & 5 & -2 \\ 4 & -6 & 1 \\ 3 & -4 & 3 \end{bmatrix}$	1
9	**Writes that B has 3 rows and 2 columns.**	0.5
	Gives reason that, since A is a 4 × 3 matrix, B must have 3 rows. Further, since the product C is a 4 × 2 matrix, B must have 2 columns.	0.5
10	**Finds A^2 as:** $A^2 = \begin{bmatrix} 1 & -6 \\ 2 & -3 \end{bmatrix}$	1

Q.No	Teacher should award marks if students have done the following:	Marks
	Writes the given equation as: $p\begin{bmatrix}1 & -6\\2 & -3\end{bmatrix}+q\begin{bmatrix}2 & -3\\1 & 0\end{bmatrix}+r\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$	**0.5**
	Writes the equation 2 p + q = 0 to find the value of ($\frac{-q}{p}$) as 2. **(Award full marks if any other valid equation is correctly used.)**	**0.5**
11	**Writes that, since B is a symmetric matrix, B = B'.**	**0.5**
	Proves that BB' is a symmetric matrix as: **(BB')' = (B')' × B' = BB'** **(Award 0.5 marks if only an example is written instead of a proof.)**	**1.5**
12	**Writes that D' = D as D is a symmetric matrix.**	**0.5**
	Writes that D' = -D as D is a skew symmetric matrix.	**0.5**
	Uses the above steps to conclude that D is a null matrix of order 3. The working may look as follows: **D = -D** **⇒ 2D = 0** **⇒ D = 0** **(Award 0.5 marks if the correct conclusion is written without any working.)**	**1**
13	**Writes that the given statement will be true if A is invertible or if A^{-1} exists.**	**1**
	Shows the working as follows: **Let AB = AC** **Pre multiplying both sides by A^{-1}, we get:** $\Rightarrow A^{-1}(AB) = A^{-1}(AC)$ $\Rightarrow (A^{-1}A)B = (A^{-1}A)C$ $\Rightarrow (I)B = (I)C$ $\Rightarrow B = C$	**1**

Q.No	Teacher should award marks if students have done the following:	Marks
14	**Writes yes.**	0.5
	Writes a justification. For example, assumes B to not be a diagonal matrix and writes an equation as: $\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}\begin{bmatrix} d_{11} & 0 \\ 0 & d_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ => $\begin{bmatrix} b_{11}d_{11} & b_{12}d_{22} \\ b_{21}d_{11} & b_{22}d_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **(Award full marks if a counter example is written.)**	1
	Uses the above equality to write that, as d_{11} and d_{22} are non-zero, b_{21} and b_{12} must be zero. Hence, concludes that B is a diagonal matrix.	0.5
15	**i) Finds the number of chips packets remaining with the distributors at the end of the month in the matrix form as:** $M - N = \begin{bmatrix} 965 & 498 \\ 872 & 689 \end{bmatrix} - \begin{bmatrix} 956 & 399 \\ 650 & 511 \end{bmatrix}$ $= \begin{array}{cc} SC & CC \end{array} \begin{bmatrix} 9 & 99 \\ 222 & 178 \end{bmatrix} \begin{array}{l} \longrightarrow \text{Distributor 1} \\ \longrightarrow \text{Distributor 2} \end{array}$	1
	ii) Finds the total cost of the chips distributed by each distributor that month using matrix multiplication as: $\begin{array}{cc} SC & CC \end{array} \begin{bmatrix} 956 & 399 \\ 650 & 511 \end{bmatrix} \times \begin{bmatrix} 10 \\ 20 \end{bmatrix} = \begin{bmatrix} 9560 + 7980 \\ 6500 + 10220 \end{bmatrix} = \begin{bmatrix} 17540 \\ 16720 \end{bmatrix} \begin{array}{l} \longrightarrow \text{Distributor 1} \\ \longrightarrow \text{Distributor 2} \end{array}$	2

Q.No	Teacher should award marks if students have done the following:	Marks
16	**Finds A^2 as:** $A^2 = \begin{bmatrix} 1 & 1 \\ 0 & \beta \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 0 & \beta \end{bmatrix} = \begin{bmatrix} 1 & 1+\beta \\ 0 & \beta^2 \end{bmatrix}$	1
	Equates A^2 to B and writes the matrix equation as: $\begin{bmatrix} 1 & 1+\beta \\ 0 & \beta^2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 9 \end{bmatrix}$ **Equates the corresponding elements and writes the equations $1 + \beta = 2$ and $\beta^2 = 9$.**	0.5
	Solves the first equation to get $\beta = 1$ and writes that $\beta = 1$ doesn't satisfy the equation $\beta^2 = 9$.	1
	Concludes that $A^2 = B$ is not possible for any value of β.	0.5
17	**Writes that by associative law of matrix multiplication $(AB) \times C = A \times (BC)$.**	0.5
	Finds A^{-1} as: $A^{-1} = \frac{1}{31}\begin{bmatrix} 5 & 3 \\ -7 & 2 \end{bmatrix}$	1
	Pre-multiplies A^{-1} on both sides of the equation $(AB) \times C = A \times (BC)$ to find BC as: **$A^{-1} \times [(AB) \times C] = A^{-1} \times [A \times (BC)] = (A^{-1} A) \times (BC) = I \times (BC) = BC$** $BC = \frac{1}{31}\begin{bmatrix} 5 & 3 \\ -7 & 2 \end{bmatrix} \times \begin{bmatrix} -13 & 12 \\ 32 & 11 \end{bmatrix}$ $\Rightarrow BC = \begin{bmatrix} 1 & 3 \\ 5 & -2 \end{bmatrix}$	1.5

Q.No	Teacher should award marks if students have done the following:	Marks
18	**i) Assumes a square matrix, B, of order 2 and writes the equation:** $$\begin{bmatrix} b_{11}+2b_{21} & b_{12}+2b_{22} \\ 2b_{11} & 2b_{12} \end{bmatrix} = \begin{bmatrix} b_{11}+2b_{12} & 2b_{11} \\ b_{21}+2b_{22} & 2b_{21} \end{bmatrix}$$	1
	Equates the corresponding terms of the matrices to conclude $b_{21} = b_{12}$.	0.5
	Equates the corresponding terms of the matrices and uses the above step to conclude $b_{22} = (2b_{11} - b_{12}) \div 2$.	0.5
	ii) Writes that C does not satisfy AC = CA as $c_{22} \neq (2c_{11} - c_{12}) \div 2$.	1
19	**Writes that since $(A^2 - A)$ is invertible, there exists a unique square matrix B of order *m* such that:** **$(A^2 - A)B = I$**	1
	Writes the above equation as: **A(A - I)B = I**	0.5
	Assumes (A - I)B as C where C is a square matrix of order *m*.	0.5
	Rewrites the equation in step 2 as: **AC = I**	0.5
	Writes that, similarly, CA = I and concludes that if, $(A^2 - A)$ is invertible, then A is invertible.	0.5
20	**Represents the quantity of sugar, wheat and rice to be purchased by Achal by the matrix:** $$\begin{bmatrix} 2 & 10 & 5 \end{bmatrix}$$	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Represents the prices of sugar, wheat and rice at the general store and the supermarket by the matrix: $\begin{bmatrix} 50 & 44 \\ 35 & 30 \\ 40 & 38 \end{bmatrix}$	**1**
	Finds the total cost at the two places as: $\begin{bmatrix} 2 & 10 & 5 \end{bmatrix} \begin{bmatrix} 50 & 44 \\ 35 & 30 \\ 40 & 38 \end{bmatrix} = \begin{bmatrix} 650 & 578 \end{bmatrix}$	**1**
	Finds Achal's total savings if he buys groceries from the supermarket as 650 - 578 - 20 = Rs 52.	**0.5**
21	**i) Writes that, the cost for each firm can be found as:** $\begin{bmatrix} 30 & 10 \\ 25 & 15 \end{bmatrix} \times \begin{bmatrix} 70 \\ 100 \end{bmatrix}$	**0.5**
	Calculates the cost for firm 1 as Rs 3100 and the cost for firm 2 as Rs 3250.	**1**
	Writes that firm 2 cost more to the company.	**0.5**
	ii) Finds the total revenue generated by each firm from the outsourcing as: $300 \times \begin{bmatrix} 500 & 200 \\ 400 & 300 \end{bmatrix}$	**0.5**
	Writes that posts from firm 1 generated 1,50,000 + 60,000 = Rs 2,10,000 in revenue and the posts from firm 2 generated 1,20,000 + 90,000 = Rs 2,10,000.	**1**

Q.No	Teacher should award marks if students have done the following:	Marks
	Calculates the profit generated by firm 1 as 2,10,000 - 3,100 = Rs 206,900 and firm 2 as 2,10,000 - 3,250 = Rs 2,06,750.	**1**
	Writes that firm 1 was more profitable for the company than firm 2.	**0.5**
22	**Finds the encrypted form of Q as:** $K \times Q = \begin{bmatrix} 3 & 7 \\ 4 & 1 \end{bmatrix} \times \begin{bmatrix} 19 \\ -20 \end{bmatrix} = \begin{bmatrix} -83 \\ 56 \end{bmatrix}$	**1**
23	**Finds the determinant of matrix K as (-25).**	**1**
	Finds the matrix used for decryption corresponding to K as: $K^{-1} = \frac{-1}{25}\begin{bmatrix} 1 & -7 \\ -4 & 3 \end{bmatrix}$	**1**
24	**Finds the original coordinates of M as:** $\frac{-1}{25}\begin{bmatrix} 1 & -7 \\ -4 & 3 \end{bmatrix} \times \begin{bmatrix} 160 \\ 105 \end{bmatrix} = \begin{bmatrix} 23 \\ 13 \end{bmatrix}$	**1**
25	**Finds K_1 as:** $\begin{bmatrix} 3 & 7 \\ 4 & 1 \end{bmatrix} \xrightarrow{R_2 \to R_2 + R_1} \begin{bmatrix} 3 & 7 \\ 7 & 8 \end{bmatrix}$	**1**

Chapter - 4
Determinants

Q: 1 $\begin{vmatrix} p & q & r \\ a & b & c \\ 2x & 2y & 2z \end{vmatrix} + 2\begin{vmatrix} p & q & r \\ d & e & f \\ x & y & z \end{vmatrix}$

Which of the following is equal to the above sum?

A. $\begin{vmatrix} 3p & 3q & 3r \\ a+2d & b+2e & c+2f \\ 4x & 4y & 4z \end{vmatrix}$

B. $\begin{vmatrix} p & q & r \\ a+d & b+e & c+f \\ 2x & 2y & 2z \end{vmatrix}$

C. $\begin{vmatrix} 2p & 2q & 2r \\ a+2d & b+2e & c+2f \\ 3x & 3y & 3z \end{vmatrix}$

D. $\begin{vmatrix} p & q & r \\ a+2d & b+2e & c+2f \\ x & y & z \end{vmatrix}$

1 A
2 B
3 C
4 D

Q: 2 **P is a matrix of order *n*.**

How many minors are there in the determinant of P?

1 (n - 1)
2 n
3 n^2
4 n^n

Q: 3 **Which of the following matrices is nonsingular?**

1 $\begin{bmatrix} -6 & 3 & 5 \\ 0 & 0 & 0 \\ 4 & 9 & -5 \end{bmatrix}$

2 $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

3 $\begin{bmatrix} 1 & 3 & 2 \\ 3 & -2 & 6 \\ 4 & 7 & 8 \end{bmatrix}$

4 $\begin{bmatrix} 0 & 2 & 5 \\ 2 & 0 & 7 \\ 5 & 7 & 0 \end{bmatrix}$

Q: 4 **S is a diagonal matrix with all the principal diagonal elements equal to *p* (*p* ≠ 0).**

What is the determinant of S^{-1} ?

1 $\frac{1}{p}$
2 $\frac{1}{p^3}$
3 $\frac{1}{p^3}\mathrm{I}$
4 p^3

Q: 5

$$|A| = \begin{vmatrix} \sin x & \square & \sin 3x \\ 3\cos x & 6\sin 2x & 9\cos 3x \\ 2\sin x & 4\cos 2x & 2\sin 3x \end{vmatrix}$$

If |A| = 0, which of the following could be ☐?

1 sin 2 *x* **2** cos 2 *x* **3** 2 cos 2 *x* **4** 4 cos 2 *x*

Q: 6 **Evaluate the determinant, |A|, of the matrix shown below. Justify your answer.** **[1]**

$$A = \begin{bmatrix} 3 & 4 & -1 \\ 2 & 5 & 0 \end{bmatrix}$$

Q: 7 **Shown below is a matrix T and its cofactor matrix, where A_{ij} is the cofactor of the element a_{ij}.** **[1]**

$$T = \begin{bmatrix} 2 & -3 & 1 \\ a_{21} & 0 & a_{23} \\ 1 & a_{32} & a_{33} \end{bmatrix}$$

$$\text{Cofactor matrix of } T = \begin{bmatrix} 8 & -9 & -10 \\ A_{21} & -3 & A_{23} \\ -12 & A_{32} & A_{33} \end{bmatrix}$$

Find the determinant of T using the given information. Show your work.

Q: 8 **P and Q are square matrices of order 4. The determinants of P and Q are 5 and 4 respectively.** **[1]**

Find the determinant of the matrix $3P^2Q$. Show your work.

Q: 9 **Shown below are two matrices A and B such that det (A) = det (B).** **[1]**

$$A = \begin{bmatrix} 2 & 0 & 6 \\ 3 & 8 & 1 \\ 1 & -7 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 3 & x \\ 0 & 8 & -7 \\ 6 & 1 & 2 \end{bmatrix}$$

Without evaluating the determinants, find the value of x. Give a valid reason.

Q: 10 **Amit and Rohan are solving a question on determinants, where β is a non-zero real number. Shown below is a part of their working.** **[1]**

$$\text{Amit}: \left| \frac{1}{\beta} \begin{bmatrix} 1 & -2 \\ 0 & \beta \end{bmatrix} \right| = \frac{\beta}{\beta} = 1$$

$$\text{Rohan}: \left| \frac{1}{\beta} \begin{bmatrix} 1 & -2 \\ 0 & \beta \end{bmatrix} \right| = \frac{\beta}{\beta^2} = \frac{1}{\beta}$$

Whose working is correct? Give a reason.

Q: 11 **[2]**

$$|A| = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \qquad |B| = \begin{vmatrix} kb_1 & kc_1 & ka_1 \\ b_2 & c_2 & a_2 \\ b_3 & c_3 & a_3 \end{vmatrix}$$

If |A| = k, where k is a real number, find |B| using the properties of determinants without expanding.

Q: 12 **If a, b and c are the first three terms of a geometric progression, find the value of ▲ without expanding the determinant.** **[2]**

$$\triangle = \begin{vmatrix} b & 2a & 3a \\ abc & abc & abc \\ c & 2b & 3b \end{vmatrix}$$

Show your steps.

Q: 13 **Given below is the adjoint of matrix B.** **[2]**

$$\begin{bmatrix} -1 & -1 & 2 \\ 1 & 4 & 3 \\ 0 & 1 & -2 \end{bmatrix}$$

Find the determinant of matrix B. Show your work with valid reasons.

Q: 14 **Shown below is a quadrilateral with A(1, 2), B(5, 1), C(4, 6) and D(2, 6) as its vertices.** **[3]**

Find the area of ABCD using determinants. Show your steps.

Q: 15 **In an arithmetic progression, the 11 th, 23 rd and 37 th terms are p, q and r respectively with a common difference of d.** **[3]**

Without expanding the determinant, find ▲.

$$\triangle = \begin{vmatrix} p & q & r \\ 11 & 23 & 37 \\ d & d & d \end{vmatrix}$$

Show your work with valid reasons.

Q: 16 **A theatre has two categories of tickets, one for adults and one for children.** **[3]**

Bindu's family paid Rs 400 to buy 6 tickets for adults and 4 tickets for children.
Nalini's family paid Rs 325 to buy 5 tickets for adults and 3 tickets for children.

Find the cost of each category of the ticket by matrix method. Show your steps.

Q: 17 **[3]**

$$f(x) = \begin{vmatrix} (x+1) & x(x+1) & x(x-1)(x+1) \\ x & x^2 & 3x^3 \\ (x-1) & x(x-1) & x(x-1)(x+1) \end{vmatrix}$$

Find f (2022). Show your steps with valid reasons.

(Hint: You need not expand the determinant.)

Q: 18 **[5]**

Given $A = \begin{bmatrix} 5 & 3 \\ -2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -3 \\ 4 & -8 \end{bmatrix}$.

Show that (adj A) × (adj B) = adj(BA).

Q: 19 **An insurance company agent has the following record of policies sold in the month of April, May and June 2022 for three different policies - Policy A, Policy B and Policy C. He is paid a fixed commission per policy sold but the commission varies for the policies A, B and C.** **[5]**

Months	Number of policies Sold			Total commission earned in the month (in Rs)
	Policy A	Policy B	Policy C	
April	8	4	6	7850
May	9	9	6	9600
June	12	9	12	15000

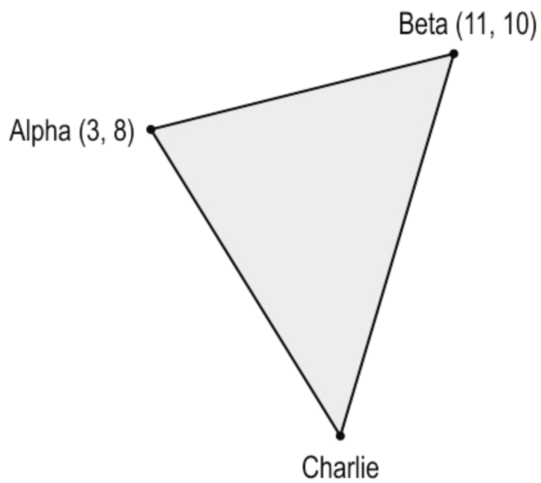
Find the fixed commission payable on policies A, B and C per unit using matrix method. Show your work.

Q: 20 **$y = ax^2 + bx + c$ passes through the points (-1, 0), (2, 12) and (3, 20).** **[5]**

Use the matrix method to find the quadratic equation. Show your steps.

Answer the questions based on the information given below.

In an army camp, three teams, Alpha, Beta and Charlie are located at the three corners of a triangular plot as shown below.



The area of the triangular plot is 37 sq units and the coordinates of team Charlie lie on the x-axis (positive direction). Team Charlie received a message from the camp head about a secret room as follows.

The coordinates of the secret room are (a_{12}, a_{21}) of the adjoint of the matrix $\begin{bmatrix} 3 & 8 \\ 11 & 10 \end{bmatrix}$.

Q: 21 **Find the *x* -coordinate of the location of team Charlie. Use the determinant method and show your steps.** **[2]**

Q: 22 **The head of the camp plans to have a medical centre on the line joining the coordinates of teams Alpha and Beta such that its *x* -coordinate is 5.** **[2]**

Find the *y* -coordinate of the medical centre using the determinant method. Show your steps and give a valid reason.

Q: 23 **Find the coordinates of the secret room. Show your steps.** **[1]**

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	2
2	3
3	4
4	2
5	3

Q.No	Teacher should award marks if students have done the following:	Marks
6	**Writes that the given determinant cannot be evaluated as only square matrices have determinants.**	1
7	**Finds the determinant of T as:** $a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} = 2(8) + (-3)(-9) + 1(-10) = 33$.	1
8	**Finds** $\lvert 3P^2Q \rvert$ **as** $3^4 \times 5 \times 5 \times 4 = 3^4(100) = 8100$.	1
9	**Writes the value of** x **as 1.**	0.5
	Writes that the value of the determinant is the same when its rows and columns are interchanged.	0.5
10	**Writes that Rohan's working is correct and gives a reason. For example, If A =** k **B, where A and B are square matrices of order** n**, then** $\lvert A\rvert = k^n \lvert B\rvert$**, where** n **= 1, 2, 3, ...**	1
11	**Mentions the property that, if each element of a row (or a column) of a determinant is multiplied by a constant** k**, then the value of the determinant gets multiplied by** k **.** **Uses the above property and writes that,** $\lvert B\rvert = \begin{vmatrix} kb_1 & kc_1 & ka_1 \\ b_2 & c_2 & a_2 \\ b_3 & c_3 & a_3 \end{vmatrix} = k\begin{vmatrix} b_1 & c_1 & a_1 \\ b_2 & c_2 & a_2 \\ b_3 & c_3 & a_3 \end{vmatrix}$	0.5
	Mentions the property that, the value of the determinant remains unchanged if its rows and columns are interchanged. **Uses the above property and writes that,** $\lvert B\rvert = k\begin{vmatrix} b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \end{vmatrix}$	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Applies row transformations on the above determinant to bring it to the form of |A|. **For example,** $\lvert \boldsymbol{B}\rvert = k\begin{vmatrix} b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \end{vmatrix} \xrightarrow{R_2 \longleftrightarrow R_3} k\begin{vmatrix} b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \xrightarrow{R_1 \longleftrightarrow R_2} k\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$	**0.5**
	Mentions the property that, if any two rows of a determinant are interchanged, the sign of the determinant changes. **Uses the above property twice and concludes that |B| = k^2.**	**0.5**
12	**Substitutes *b* and *c* in the determinant with *ar* and ar^2 respectively, where *a* is the first term and *r* is the common ratio of the geometric progression and rewrites the given determinant as:** $\triangle = \begin{vmatrix} ar & 2a & 3a \\ a^3r^2 & a^3r^2 & a^3r^2 \\ ar^2 & 2ar & 3ar \end{vmatrix}$	**0.5**
	Uses the properties and evaluates the determinant as: $\triangle = (a)\,(a^3r^2)\,(ar)\begin{vmatrix} r & 2 & 3 \\ 1 & 1 & 1 \\ r & 2 & 3 \end{vmatrix}$ $\triangle = (a^5)\,(r^3)\,(0)$ $\triangle = 0$	**1.5**
13	**Writes that for a square matrix of order n, |adj B| = |B| $^{(n-1)}$.** **Applies the above result on (adj B) and writes |adj B| = |B|^2.**	**1**

Q.No	Teacher should award marks if students have done the following:	Marks
	Finds |adj B| as: **(-1)(-8 - 3) - 1(2 - 2) = 11**	**0.5**
	Uses the relation in step 1 and finds |B| as √11 or (-√11).	**0.5**
14	**Divides the quadrilateral into two triangles ABC and ACD and writes an expression for the area of the quadrilateral using determinants. For example:** $\frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 5 & 1 & 1 \\ 4 & 6 & 1 \end{vmatrix} + \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 4 & 6 & 1 \\ 2 & 6 & 1 \end{vmatrix}$	**1**
	Simplifies the above expression by evaluating the determinants as $\frac{1}{2}$ **[1(1 - 6) - 2(5 - 4) + 1(30 - 4)] +** $\frac{1}{2}$ **[1(6 - 6) - 2(4 - 2) + 1(24 - 12)].**	**1**
	Finds the area of the quadrilateral as $\frac{27}{2}$ **sq units.**	**1**
15	**Takes the first term of the arithmetic progression as *a* and rewrites the given determinant ▲ as:** $\triangle = \begin{vmatrix} a + 10d & a + 22d & a + 36d \\ 11 & 23 & 37 \\ d & d & d \end{vmatrix}$	**0.5**
	Uses the properties of determinants and rewrites the above determinant as: $\triangle = \begin{vmatrix} a & a & a \\ 11 & 23 & 37 \\ d & d & d \end{vmatrix} + \begin{vmatrix} 10d & 22d & 36d \\ 11 & 23 & 37 \\ d & d & d \end{vmatrix}$	**0.5**

Q.No	Teacher should award marks if students have done the following:	Marks
	Uses the properties of determinants and rewrites the above determinant as: $$\triangle = ad\begin{vmatrix} 1 & 1 & 1 \\ 11 & 23 & 37 \\ 1 & 1 & 1 \end{vmatrix} + d^2\begin{vmatrix} 10 & 22 & 36 \\ 11 & 23 & 37 \\ 1 & 1 & 1 \end{vmatrix}$$	**0.5**
	Writes that in the first determinant R_1 and R_3 are identical hence the value of the determinant is 0.	**0.5**
	Writes that in the second determinant $R_2 = R_1 + R_3$ hence the value of the determinant is 0.	**0.5**
	Concludes that $\triangle = ad(0) + d^2(0) = 0$.	**0.5**
16	**Assumes the cost of each adult ticket as x and each child's ticket as y. Writes the equations that represent the given scenario as follows:** $6x + 4y = 400$ $5x + 3y = 325$	**0.5**
	Writes the above system of equations in the matrix form as AX = B where: $$A = \begin{bmatrix} 6 & 4 \\ 5 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 400 \\ 325 \end{bmatrix}$$	**0.5**
	Finds |A| as 18 - 20 = -2 ≠ 0. Hence writes that A^{-1} exists and the system has a unique solution.	**0.5**
	Finds A^{-1} as: $A^{-1} = \frac{1}{\|A\|} \times \text{adj } A$ $$\Rightarrow \frac{1}{-2}\begin{bmatrix} 3 & -4 \\ -5 & 6 \end{bmatrix}$$	**0.5**

Q.No	Teacher should award marks if students have done the following:	Marks
	Writes that X = A^{-1} B and finds X as: $X = \frac{1}{-2}\begin{bmatrix} 3 & -4 \\ -5 & 6 \end{bmatrix}\begin{bmatrix} 400 \\ 325 \end{bmatrix}$ $\Rightarrow X = \begin{bmatrix} 50 \\ 25 \end{bmatrix}$ **Concludes that the cost of each adult and child's ticket is Rs 50 and Rs 25 respectively.**	1
17	**Takes (*x* + 1), *x* and (*x* - 1) common from the first, second and third rows respectively to write:** $f(x) = x\,(x+1)\,(x-1)\begin{vmatrix} 1 & x & x\,(x-1) \\ 1 & x & 3\,x^2 \\ 1 & x & x\,(x+1) \end{vmatrix}$	1
	Takes *x* common from the second column to write: $f(x) = x^2\,(x+1)\,(x-1)\begin{vmatrix} 1 & 1 & (x-1)\,x \\ 1 & 1 & 3\,x^2 \\ 1 & 1 & x\,(x+1) \end{vmatrix}$	0.5
	Writes that the first and second columns are identical. Hence, concludes that *f* (*x*) = 0 for any value of *x* . **(Award only 0.5 marks if the reason is not mentioned.)**	1

Q.No	Teacher should award marks if students have done the following:	Marks
	Uses the above step to conclude that *f* (2022) = 0.	**0.5**
18	**Finds adj A as:** adj A = $\begin{bmatrix} 4 & -3 \\ 2 & 5 \end{bmatrix}$	**1**
	Finds adj B as: adj B = $\begin{bmatrix} -8 & 3 \\ -4 & 2 \end{bmatrix}$	**1**
	Finds (adj A) × (adj B) as: adj A × adj B = $\begin{bmatrix} -20 & 6 \\ -36 & 16 \end{bmatrix}$	**1**
	Finds BA as: B × A = $\begin{bmatrix} 16 & -6 \\ 36 & -20 \end{bmatrix}$	**1**
	Finds adj(BA) as: adj (BA) = $\begin{bmatrix} -20 & 6 \\ -36 & 16 \end{bmatrix}$ **Uses step 3 and step 5 to conclude that (adj A) × (adj B) = adj(BA).**	**1**

Q.No	Teacher should award marks if students have done the following:	Marks
19	**Takes the fixed commission payable on policies A, B and C per unit as x, y and z respectively and frames the system of linear equations as:** $8x + 4y + 6z = 7850$ $9x + 9y + 6z = 9600$ $12x + 9y + 12z = 15000$ **(Award full marks if a student skips this step and writes the system in matrix form directly.)**	0.5
	Writes the system of equations in the form of a matrix equation as AX = B, where, $A = \begin{bmatrix} 8 & 4 & 6 \\ 9 & 9 & 6 \\ 12 & 9 & 12 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 7850 \\ 9600 \\ 15000 \end{bmatrix}$	0.5
	Finds $\lvert A\rvert = 126 \neq 0$ and concludes that A^{-1} exists.	0.5
	Finds adjA as: $\text{adj } A = \begin{bmatrix} 54 & 6 & -30 \\ -36 & 24 & 6 \\ -27 & -24 & 36 \end{bmatrix}$	1.5
	Finds A^{-1} as: $A^{-1} = \frac{1}{\lvert A\rvert} = \text{adj } A = \frac{1}{126}\begin{bmatrix} 54 & 6 & -30 \\ -36 & 24 & 6 \\ -27 & -24 & 36 \end{bmatrix}$	0.5

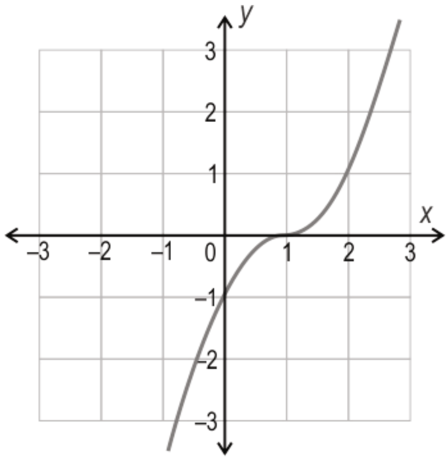
Q.No	Teacher should award marks if students have done the following:	Marks
	Rewrites the matrix equation in step 2 as X = A $^{-1}$ B and solves the same to obtain the values of x, y and z as 250, 300 and 775 respectively as: $X = A^{-1}B = \frac{1}{126}\begin{bmatrix} 54 & 6 & -30 \\ -36 & 24 & 6 \\ -27 & -24 & 36 \end{bmatrix}\begin{bmatrix} 7850 \\ 9600 \\ 15000 \end{bmatrix} = \begin{bmatrix} 250 \\ 300 \\ 775 \end{bmatrix}$ **Hence concludes that the fixed commission payable on policies A, B and C per unit are Rs 250, Rs 300 and Rs 775 respectively.**	**1.5**
20	**Writes the system of equations as:** $a - b + c = 0$ $4a + 2b + c = 12$ $9a + 3b + c = 20$	**0.5**
	Writes the above system of equations in the form AX = B as: $\begin{bmatrix} 1 & -1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \\ 20 \end{bmatrix}$	**0.5**
	Finds |A| as 1(2 - 3) + 1(4 - 9) + 1(12 - 18) = -12 and writes that A $^{-1}$ exists as |A| ≠ 0.	**0.5**
	Finds A $^{-1}$ as: $A^{-1} = \frac{-1}{12}\begin{bmatrix} -1 & 4 & -3 \\ 5 & -8 & 3 \\ -6 & -12 & 6 \end{bmatrix}$ **(Award 1 mark if only all the cofactors are found correctly.)**	**2**

Q.No	Teacher should award marks if students have done the following:	Marks
	Writes X as: $$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \frac{-1}{12}\begin{bmatrix} -1 & 4 & -3 \\ 5 & -8 & 3 \\ -6 & -12 & 6 \end{bmatrix}\begin{bmatrix} 0 \\ 12 \\ 20 \end{bmatrix}$$	**0.5**
	Finds the values of a, b and c as 1, 3 and 2 and finds the quadratic equation as $x^2 + 3x + 2$.	**1**
21	**Uses the determinant method to write the expression for the area of a triangle as:** $$\frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ 11 & 10 & 1 \\ x & 0 & 1 \end{vmatrix} = 37$$	**0.5**
	Expands the LHS of the above equation and simplifies it to get: $-58 - 2x = \pm 74$	**1**
	Finds the value of x as (-66) or 8 and concludes that $x = 8$.	**0.5**
22	**Writes that the Alpha team, medical centre and the Beta team are on the same line and hence the area of the triangle formed by three collinear points will be zero.**	**0.5**
	Writes the relation as: $$\frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ 11 & 10 & 1 \\ 5 & y & 1 \end{vmatrix} = 0$$	**0.5**
	Simplifies the above equation to find the value of the y -coordinate as $\frac{68}{8}$ or 8.5 units.	**1**

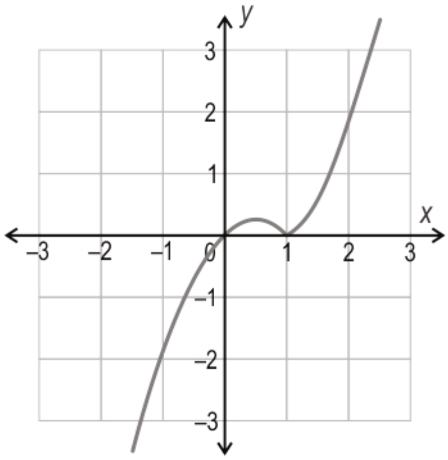
Q.No	Teacher should award marks if students have done the following:	Marks
23	Finds the adjoint of the given matrix as: $\begin{bmatrix} 10 & -8 \\ -11 & 3 \end{bmatrix}$	0.5
	Writes the coordinates of the location of the secret room as (-11, -8).	0.5

Chapter - 5
Continuity and Differentiability

Q: 1 **Shown below are the graphs of two functions.**



$g(x) = (x - 1)\,|x - 1|$



$f(x) = x\,|x - 1|$

What can one conclude from the above graphs?

1 Product of a differentiable function and a non-differentiable function is ALWAYS differentiable.
2 Product of a differentiable function and a non-differentiable function is ALWAYS NOT differentiable.
3 Product of a differentiable function and a non-differentiable function MAY BE differentiable.
4 (cannot conclude anything from the given graphs.)

Q: 2 **For what real value of α, is the function given below continuous for $x \in (-\infty, +\infty)$?**

$$f(x) = \begin{cases} \dfrac{1 - \cos 4x}{\alpha x^2} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$

1 2
2 4
3 8
4 (such a value of α does not exist)

Q: 3 **Which of the following is INCORRECT about a function $f : R \to R$?**

1 If f is differentiable at $x = c$, then f is continuous at $x = c$.
2 If f is not differentiable at $x = c$, then f is not continuous at $x = c$.
3 If f is not continuous at $x = c$, then f is not differentiable at $x = c$.
4 If f is continuous at $x = c$, then f may or may not be differentiable at $x = c$.

Q: 4 **In which of these sets is the function $f(x) = x\,|x - 2|^2$ differentiable twice?**

1 R
2 R - {2}
3 R - {0, 2}
4 (the function cannot be differentiated twice in R)

Q: 5 **[1]**

If $f(x) = \sqrt{\cos^2 x - 25}$, then $f'(x) = \dfrac{1}{2\sqrt{\cos^2 x - 25}}g(x)$.

Find $g(x)$. Show your steps.

Q: 6 **A teacher asked her students for an example of a function whose first-order derivative is the same as its second-order derivative.** **[1]**

Shyama said, "there is no such function".

Is Shyama correct? Justify your answer.

Q: 7 **Differentiate $y = e^{\log \sin x}$, where $x \in (0, \pi)$, with respect to x. Show your steps.** **[1]**

Q: 8 **Is the following statement true or false? Justify your answer.** **[1]**

Statement: $\sin \frac{1}{x}$ is a continuous function in the domain of real numbers.

Q: 9 **[1]**

Dhriti was asked to check the continuity of the function $f(x) = \dfrac{x^2 - 2x}{x^2 - 4}$ at $x = 2$.
She simplified the limit as show below:

Left hand limit: $\lim\limits_{x \to 2^-} \dfrac{x^2 - 2x}{x^2 - 4} = \lim\limits_{x \to 2^-} \dfrac{x(x-2)}{(x+2)(x-2)} = \lim\limits_{x \to 2^-} \dfrac{x}{(x+2)} = \dfrac{2}{2+2} = \dfrac{1}{2}$

Right hand limit: $\lim\limits_{x \to 2^+} \dfrac{x^2 - 2x}{x^2 - 4} = \lim\limits_{x \to 2^+} \dfrac{x(x-2)}{(x+2)(x-2)} = \lim\limits_{x \to 2^+} \dfrac{x}{(x+2)} = \dfrac{2}{2+2} = \dfrac{1}{2}$

She then concluded that $f(x)$ is continuous at $x = 2$.

Is Dhriti's conclusion correct? Justify your answer.

Q: 10 **Examine if Rolle's theorem is applicable to the following function.** **[2]**

$$f(x) = \frac{(x-1)^3}{15} - \frac{(x-1)^2}{5}\ ; x \in [1, 4]$$

Show your steps and give a valid reason.

Q: 11 **Manu and Madhura were asked to examine if Rolle's theorem is applicable to the following function:** **[2]**

$$f(x) = 1 + \sqrt[3]{x^2}\text{, for } x \in [-1, 1]$$

Manu concluded that Rolle's theorem is applicable and Madhura concluded that Rolle's theorem is not applicable to the given function.

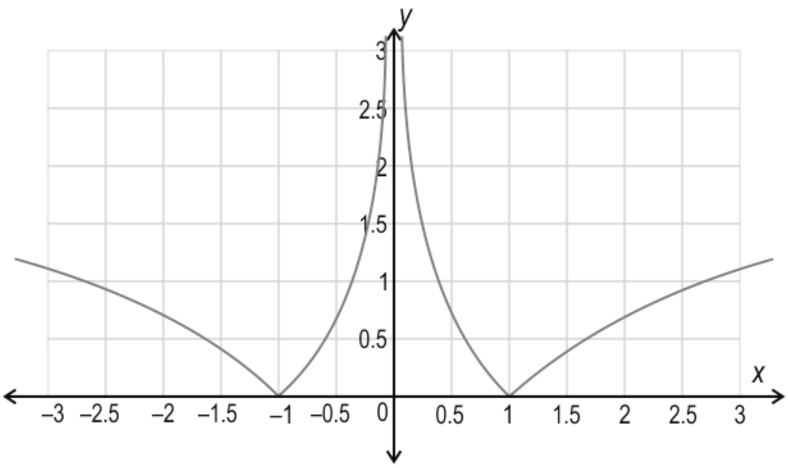
Who is correct? Justify your answer.

Q: 12 **Differentiate the following function with respect to *x*.** **[2]**

$$y = \frac{e^x}{(1 + \frac{1}{1!} + \frac{1}{2!} + ...) \log x}$$

Show your steps.

Q: 13 **Shown below is the graph of a function discontinuous at *x* = 0.** **[2]**



Identify the point(s) in the domain [-3, 3], where the function is NOT differentiable. Give a reason for each point of non-differentiability.

Q: 14 **If x = a(log t) and $f(t)$ = a($\sin^{-1} t$) where a is a constant, find $\frac{df(t)}{dx}$. Show your steps.** **[2]**

Q: 15 **At 4:30 p.m., a boat was travelling at a velocity of 28 km/h. Three minutes later, the boat was travelling at a velocity of 92 km/h.** **[3]**

Based on the above information, state whether the following assertion is true or false. Justify your answer.

"The acceleration was 1280 km/h^2 at some point during the 3 minute interval."

Q: 16 **Find the subset of R in which the function *f* (*x*) = *x* | *x* | is differentiable twice. Show your work with a valid reason.** **[3]**

Q: 17 **Differentiate:** **[3]**

$$y = \cos^{-1}\frac{(1+3^x)(1-3^x)}{1+3^{2x}}$$

Show your steps.

Q: 18 **Check if the mean value theorem holds for the function *f* (*x*) = √(4 - *x*) in the interval [-5, 4]. Show your steps.** **[5]**

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	3
2	3
3	2
4	1

Q.No	Teacher should award marks if students have done the following:	Marks
5	**Uses the chain rule to find *f* '(*x*) as follows:** $$f'(x) = \frac{1}{2\sqrt{\cos^2 x - 25}} \times (-2 \sin x \cos x)$$	0.5
	Writes that *g* (*x*) = (-2 sin *x* cos *x*) or (-sin 2 *x*).	0.5
6	**Writes that Shyama is wrong.**	0.5
	Justifies by giving an example of a function whose first-order derivative is the same as its second-order derivative. **For example, *f* (*x*) = e^x.**	0.5
7	**Takes logarithm to the base *e* on both sides and simplifies the given equation as:** ***y* = sin *x***	0.5
	Differentiates the above equation with respect to *x* as: $\frac{dy}{dx}$ **= cos *x*** **(Award full marks if *y* (cot *x*) is obtained instead of cos *x* .)**	0.5
8	**Writes False(F).**	0.5
	Writes that the function is not continuous in the domain of real numbers since $\frac{1}{x}$ is not defined at *x* = 0.	0.5
9	**Writes no.**	0.5
	Writes that *f* (*x*) is not continuous at *x* = 2 because *f* (2) = $\frac{0}{0}$ which is not defined.	0.5
10	**Writes that *f* (*x*) is continuous and differentiable everywhere as *f* (*x*) is a polynomial.**	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Finds *f* (1) and *f* (4) as 0 and writes: $f(1) = f(4) = 0$	1
	Uses steps 1 and 2 to conclude that Rolle's theorem is applicable to the given function.	0.5
11	**Writes that Madhura is correct.**	0.5
	Differentiates the function as: $f^{\prime}(x) = \frac{2}{3} \times \frac{1}{\sqrt[3]{x}}$	0.5
	Writes that the function is not differentiable at *x* = 0 with 0 belonging to [-1, 1]. Hence, concludes that Rolle's theorem cannot be applied to the given function.	1
12	**Rewrites the function as:** $y = \frac{e^x}{e.\log x}$	0.5
	Differentiates the function using the quotient rule to get: $\frac{dy}{dx} = e^{x-1}\left(\frac{x(\log x) - 1}{x(\log x)^2}\right)$	1.5
13	**Identifies the points in the domain [-3, 3] where the function is not differentiable as *x* = 0, 1 and (-1).**	1
	Writes that, as the function is not continuous at *x* = 0, it is not differentiable at *x* = 0.	0.5
	Writes that at *x* = 1 and (-1), the graph is pointed/not smooth, hence not differentiable.	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
14	**Finds the derivative of *x* with respect to *t* as, $\frac{dx}{dt} = \frac{a}{t}$.**	**0.5**
	Finds the derivative of *f* (*t*) with respect to *t* as follows: $$\frac{df(t)}{dt} = \frac{a}{\sqrt{(1-t^2)}}$$	**0.5**
	Uses above steps to find $\frac{df(t)}{dx}$ as follows: $$\frac{df(t)}{dx} = \frac{t}{\sqrt{(1-t^2)}}$$	**1**
15	**Converts 3 mins to hours as:** **3 mins = $\frac{3}{60}$ hours = $\frac{1}{20}$ hours**	**0.5**
	Writes that the mean value theorem guarantees that at some time (*c*) during this 3-minute interval, the boat's acceleration, *v* '(*c*), was: $$v'(C) = \frac{v\left(\frac{1}{20}\right) - v(0)}{\frac{1}{20} - 0}, \text{ where } C \in \left(0, \frac{1}{20}\right)$$	**1**
	Substitutes the values in the above equation to get *v* '(*c*) as: ***v* '(*c*) = 20 [92 - 28] = 1280 km/h^2**	**1**
	Concludes that, as the derivative of velocity at a point is nothing but acceleration at that point, the acceleration was 1280 km/h^2 at some point during the 3 minute interval.	**0.5**

Q.No	Teacher should award marks if students have done the following:	Marks
16	**Rewrites the given function as:** $f(x)=\begin{cases}-x^2 & x<0\\ 0 & x=0\\ x^2 & x>0\end{cases}$	1
	Finds the first derivative of the given function as: $f'(x)=\begin{cases}-2x & x<0\\ 0 & x=0\\ 2x & x>0\end{cases}$	0.5
	Finds the second derivative of the given function as: $f''(x)=\begin{cases}-2 & x<0\\ 0 & x=0\\ 2 & x>0\end{cases}$	0.5
	Writes that, since $f''(0^-) \neq f''(0^+)$, $f''(0)$ does not exist.	0.5
	Finds the subset of R in which the function $f(x) = x\|x\|$ is differentiable twice as R - {0}.	0.5
17	**Rewrites the expression as:** $y = \cos^{-1}\dfrac{1-3^{2x}}{1+3^{2x}}$ **and puts 3^x as tan *t*.**	1

Q.No	Teacher should award marks if students have done the following:	Marks
	Write the expression as: $y = \cos^{-1}\dfrac{1-\tan^2 t}{1+\tan^2 t}$ or $\cos^{-1}(\cos 2t)$	0.5
	Writes that, $y = 2t$ or $y = 2\tan^{-1} 3^x$.	0.5
	Finds $\dfrac{dy}{dx}$ as $\dfrac{2}{1+(3^x)^2} \times (3^x \ln(3))$.	1
18	**Writes that $f(x) = \sqrt{(4-x)}$ is well defined for every $x \in [-5, 4]$.** **Writes that, since $f(x)$ is a root of a polynomial, it is continuous in [-5, 4].**	1
	Finds the derivative of $f(x)$ as: $f'(x) = \frac{-1}{2\sqrt{(4-x)}}$	1
	Writes that $f'(x)$ is defined in the interval (-5, 4) and hence $f(x)$ is differentiable in (-5, 4).	0.5
	Applies the mean value theorem to obtain $f'(c)$ as: $f'(c) = \dfrac{f(4)-f(-5)}{4-(-5)} = \dfrac{0-3}{9} = \dfrac{-1}{3}$, where $c \in (-5, 4)$	1
	Equates $f'(x)$ at $x = c$ with $f'(c)$ to obtain c as: $\frac{-1}{2\sqrt{(4-c)}} = \frac{-1}{3}$ $\Rightarrow c = \frac{7}{4}$	1

Q.No	Teacher should award marks if students have done the following:	Marks
	Concludes that, at $c = \frac{7}{4} \in (-5, 4)$, we have $f'(c) = \frac{-1}{3}$. Hence, the mean value theorem holds for the function $f(x) = \sqrt{4 - x}$ in the interval [-5, 4].	**0.5**

Chapter - 6
Application of Derivatives

Q: 1 **The diagonal of a square, of side $3\sqrt{2}$ cm, is increasing at a rate of 2 cm/s.**

Which of the following is the rate at which its area is increasing?

1. $\sqrt{2}$ cm^2/s
2. $6\sqrt{2}$ cm^2/s
3. 12 cm^2/s
4. 24 cm^2/s

Q: 2 **Sameer wants to find the area of his garden. He measures the side of his square garden as 10 m with an error of 0.05 m.**

Which of the following is the approximate error that Sam will have in calculating its area?

1. 0.05 m^2
2. 0.5 m^2
3. 1 m^2
4. 10.05 m^2

Q: 3 **Gagan wants to calculate the capacity of a cylindrical water tank. He precisely measures the height of the tank as 7 m. Next, he measures the radius of the tank as 2 m, with an error of 0.05 m. The error occured while measuring the radius results in an error in calculating the capacity.**

What is the approximate error in calculating the capacity of the water tank?

(Note: Take $\pi = \frac{22}{7}$.)

1. 2.2 m^3
2. 4.4 m^3
3. 44 m^3
4. 88 m^3

Q: 4 **Study the function below.**

$$f(x) = \frac{2x^3}{3} - 18x + k, \text{ where } k \text{ is a constant}$$

Which of the following is true about the nature of the function in the interval [-3, 3]?

1. $f(x)$ is increasing
2. $f(x)$ is decreasing
3. $f(x)$ is neither increasing nor decreasing
4. (cannot say without knowing the value of k)

Q: 5 **The normal to the curve $y = f(x)$ at the point (5, 7) makes an angle of $\frac{\pi}{4}$ with the x-axis in the positive direction.** **[1]**

Find $f'(5)$. Show your work.

Q: 6 **Aditi is making a circular dosa. She is spreading the dosa batter such that its radius is increasing at a rate of 2 cm/s.** **[1]**

Find the rate of change of the area of the dosa, in terms of π, when its radius is 9 cm. Show your steps.

(Note: Assume that the dosa has negligible thickness.)

Q: 7 **Find the critical points of the function $y = \tan^{-1}(\sec x)$ where $x \in [\frac{-\pi}{2}, \frac{\pi}{2}]$. Show your steps.** **[2]**

Q: 8 **The maximum value of the function $f(x) = x^{\frac{1}{x}}$, $x > 0$, is obtained at $x = e$.** **[2]**

Use the above fact and show that $e^{\pi} > \pi^{e}$.

Q: 9 **A circular metallic plate is expanding such that its area is constantly increasing with respect to time.** **[2]**

Milind claims that the rate of increase of its perimeter with respect to time is inversely proportional to its radius.

Is Milind's claim correct? Justify your answer.

Q: 10 **A particle is moving such that its distance s from a fixed point at any time t is given by $s = A\sin t + B\cos t$, where A, B ∈ R.** **[2]**

Show that the particle's acceleration is always numerically equal to its distance from the fixed point.

Q: 11 **A solid sphere of gold, of radius 5 cm, is being melted in a furnace such that the radius is decreasing uniformly.** **[2]**

When its radius is 1 cm, show that the rate at which its surface area is decreasing with respect to time is twice the rate at which its volume is decreasing with respect to time.

Q: 12 A basketball has a tiny hole that leads to it getting deflated while maintaining a spherical shape. **[3]**

Find the ratio of the rate of loss of the volume of air to the rate of loss of the surface area of the ball when the radius of the ball is 8 cm. Show your steps.

Q: 13 A cylindrical disk of radius R and height H is pressed by a hydraulic press. During the process, the radius and the height of the disk change such that the cylindrical shape is retained and the volume of the disk remains constant. **[3]**

What is the ratio of the rate of change of height to the rate of change of radius in terms of R? Show your steps and give valid reasons.

Q: 14 Companies issue shares as a means to raise money. This may be to finance company expansion, a new development, or to move into overseas markets. When you buy shares, you effectively become a part owner of the company. **[3]**

Kabir observes the price of his share in the stock market for 16 months. He comes up with a function $p(t) = 16t - t^2 + 8$ where p is the maximum price of the share (in Rs) during a month and t is the number of months since he started observing it.

Find the maximum price of the share (in Rs) during the period Kabir observed it. Show your steps and give valid reasons.

Q: 15 Simran cuts a metallic wire of length a m into two pieces. She uses both pieces to create two squares of different side lengths. **[3]**

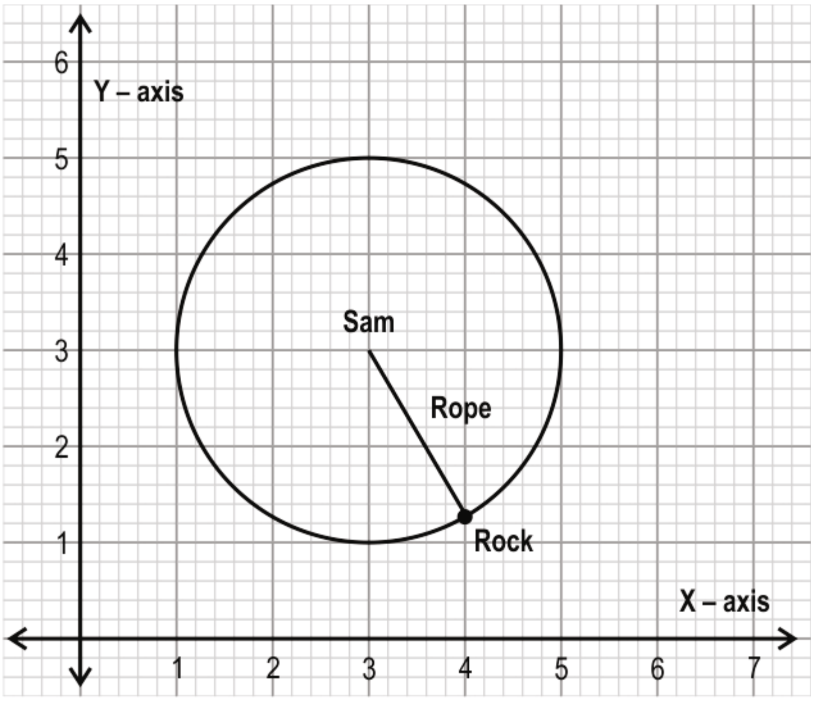
Find the side lengths of the squares (in terms of a) for which the combined area will be MINIMUM? Show your steps and give reasons.

Q: 16 The function shown below is increasing in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right]$: **[3]**

$$f(x) = \sin^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right)$$

What is the nature of the function in the interval $\left(\frac{\pi}{2}, \pi\right)$? Show your work.

Q: 17 Sameer has a piece of rock tied with a rope. He holds the other end of the rope and starts rotating the rock in a circular motion. The equation of the circular path traced by the rock is given by $x^2 + y^2 - 6x - 6y + 14 = 0$. **[3]**



When the tension reaches its maximum, the rope snaps and the rock starts moving along a straight-line path. The rock is at (4,1.27) at this point.

Find the equation of the straight-line path that the rock follows. Show your steps with valid reasons.

Q: 18 $y = x^2 + bx - b$, where b is a real number, represents a curve. **[5]**

The tangent to the curve y, at the point (1, 1), forms a triangle with the coordinate axes in the first quadrant. The area of the triangle is 2 sq units.

Find the value of b. Show your work.

Q: 19 **The life span of a certain flowering plant is around 6 years. *t* years after the sapling is planted, the plant produces *r* grams of flowers each day.** **[5]**

The relation between *r* and *t* can be approximated as:

$$r = \frac{t^3}{3} - 6t^2 + 32t,\ 0 \leq t \leq 6$$

i) What will be the yield per day, 3 years after planting the sapling?
ii) When will the maximum yield per day be obtained?

Show your steps with valid reasons.

Q: 20 **Find the equation of the tangent to the curve represented by *x* = 3 sec θ, *y* = 3 tan θ where 0 < θ < $\frac{\pi}{2}$ such that the line represented by -2 *y* = *x* + 1 is normal to the tangent at the point of contact with the curve. Show your steps.** **[5]**

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	3
2	3
3	2
4	2

Q.No	Teacher should award marks if students have done the following:	Marks
5	Finds the slope of the normal to the curve $y = f(x)$ at the point (5, 7) as tan $\frac{\pi}{4}$ = 1.	0.5
	Finds the slope of the tangent to the curve $y = f(x)$ at the point (5, 7) as $\left(\frac{-1}{1}\right) = (-1)$ and concludes that $f'(5) = (-1)$.	0.5
6	Finds the expression for the rate of change of the area of the dosa as: $\frac{dA}{dt} = \pi \times 2r \times \frac{dr}{dt}$ cm^2/s	0.5
	Uses the given information and finds the rate of change of the area of the dosa when its radius is 9 cm as $\frac{dA}{dt} = \pi \times 2 \times 9 \times 2 = 36\pi$ cm^2/s.	0.5
7	Differentiates the function as: $\frac{dy}{dx} = \frac{\sec x \tan x}{1+\sec^2 x}$	1
	Writes that 0 is a critical point as $\frac{dy}{dx}$ is 0 when $x = 0$.	0.5
	Writes that $\frac{-\pi}{2}$ and $\frac{\pi}{2}$ are critical points as $\frac{dy}{dx}$ is not differentiable at these two points.	0.5
8	Writes that, as the maximum value of $f(x)$ is obtained at $x = e$, $f(e) > f(x)$, for every $x > 0$.	0.5
	Uses the above step to write: For x = π, f(e) > f(π) $\Rightarrow e^{\frac{1}{e}} > \pi^{\frac{1}{\pi}}$	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Raises the power on both sides of the above inequality by (*e* π) and simplifies the same to get: $\left(e^{\frac{1}{e}}\right)^{e\pi} > \left(\pi^{\frac{1}{\pi}}\right)^{e\pi}$ $\Rightarrow e^{\pi} > \pi^{e}$	**1**
9	**Finds the rate of change of area of the circular plate with respect to time and equates it to a constant *k* as:** $\frac{dA}{dt} = 2\pi r \times \frac{dr}{dt} = k$ **cm^2/s, where *k* is a positive real number, *t* is the time, *A* and *r* are area and radius of the circular plate respectively.**	**0.5**
	Finds the rate of change of perimeter of the circular plate with respect to time as: $\frac{dP}{dt} = 2\pi \times \frac{dr}{dt}$ **, where *P* is the perimeter of the circular plate.**	**0.5**
	Uses steps 1 and 2 to write: $\frac{dP}{dt} = 2\pi \times \frac{k}{2\pi r} = \frac{k}{r}$	**0.5**
	Concludes that Milind's claim is correct.	**0.5**
10	**Differentiates *s* to find velocity as:** $s' = A\cos t - B\sin t$	**0.5**
	Differentiates *s*' to find acceleration as: $s'' = -A\sin t - B\cos t$	**0.5**
	Rewrites acceleration in terms of distance as $s'' = (-s)$ **and writes that the magnitudes of distance and acceleration are the same.** **Hence, concludes that acceleration is always numerically equal to the distance of the particle from the fixed point.**	**1**

Q.No	Teacher should award marks if students have done the following:	Marks
11	Writes that the volume of a sphere, V, is given by $\frac{4}{3}\pi r^3$, where r is the radius of the sphere and differentiates the same with respect to time as: $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$	0.5
	Writes that the surface area of a sphere, A, is given by $4\pi r^2$ and differentiates the same with respect to time as: $\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$	0.5
	Uses steps 1 and 2 to find the relation between $\frac{dV}{dt}$ and $\frac{dA}{dt}$ as: $\frac{dV}{dt} = \frac{r}{2} \times \frac{dA}{dt}$	0.5
	Substitutes $r = 1$ in the above equation to show that $2\frac{dV}{dt} = \frac{dA}{dt}$.	0.5
12	Writes that the volume of the air in the basketball is $\frac{4}{3}\pi r^3$ and finds the rate of loss of volume of air $\frac{dV}{dt}$ as $4\pi r^2 \frac{dr}{dt}$.	1
	Writes that the surface area of the basketball is $4\pi r^2$ and finds the rate of loss of the surface area $\frac{dA}{dt}$ as $8\pi r \frac{dr}{dt}$.	1
	Uses steps 1 and 2 to find the ratio as r : 2 and evaluates the same at r = 8 cm as 8 : 2 or 4 : 1.	1
13	Writes the volume function of the disk as $V = \pi \times R^2 \times H$.	0.5
	Writes that since volume remains constant, the derivative of V should be zero. $\frac{dV}{dt} = 0$	0.5
	Applies chain rule to get the equation as: $\pi \times R^2 \times \frac{dH}{dt} + \pi \times H \times 2R \times \frac{dR}{dt} = 0$	1
	Simplifies the equation in step 3 to get the ratio as: $\frac{dH}{dt} : \frac{dR}{dt} = \frac{-2H}{R}$	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Substitutes $H = \frac{V}{\pi R^2}$ from step 1 to get the ratio in terms of R as: $\frac{dH}{dt} : \frac{dR}{dt} = \frac{-2V}{\pi R^3}$	0.5
14	**Finds the first derivative of the function as $\frac{dp}{dt} = 16 - 2t$.**	0.5
	Calculates the critical point as $t = 8$ by equating $16 - 2t$ to 0.	0.5
	Finds the second derivative as: $\frac{d^2p}{dt^2} = -2$ **Writes that this means that the function will be at its maximum at $t = 8$.**	1
	Finds the maximum price of the share as $p(8)$ as $16 \times 8 - 8^2 + 8 =$ Rs 72.	1
15	**Assumes the perimeter of one square as x m and the perimeter of the other square as $(a - x)$ m.**	0.5
	Finds the combined area of the two squares as: $A = (\frac{x}{4})^2 + (\frac{a-x}{4})^2$ m^2	0.5
	Differentiates the combined area as: $\frac{dA}{dx} = \frac{(4x-2a)}{16}$	0.5
	Equates $\frac{dA}{dx}$ to 0 to find the critical point as $x = \frac{a}{2}$ m.	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Finds the second derivative as: $$\frac{d^2A}{dx^2} = \frac{1}{4}$$ **Concludes that $x = \frac{a}{2}$ m is a minima.**	**0.5**
	Writes that the combined area of the two squares will be minimum when the side lengths of both the squares is $\frac{a}{8}$ m.	**0.5**
16	**Simplifies the given function as:** $$f(x) = \sin^{-1}\left(\sin x \left(-\cos\frac{3\pi}{4}\right) + \cos x \sin\frac{3\pi}{4}\right)$$ **as $\sin x = \frac{1}{\sqrt{2}}$ and $-\cos x = \frac{1}{\sqrt{2}}$ only when $x = \frac{3\pi}{4}$ in the interval ($\frac{\pi}{2}$, π).**	**1**
	Simplifies the above step as: $$f(x) = \sin^{-1}\left(\sin\left(\frac{3\pi}{4} - x\right)\right) = \left(\frac{3\pi}{4} - x\right)$$	**1**
	Differentiates the above function to get $f'(x) = -1$ and concludes that the function is decreasing in the interval ($\frac{\pi}{2}$, π).	**1**
17	**Differentiates the equation of the circle to get $\frac{dy}{dx}$ as $\frac{(6-2x)}{(2y-6)}$ or $\frac{(3-x)}{(y-3)}$.**	**1**
	Calculates the slope of the curve at (4, 1.27) as $\frac{dy}{dx} = \frac{100}{173}$.	**1**
	Writes the equation of the path of the rock as $y - 1.27 = \frac{100}{173}(x - 4)$.	**1**
18	**Differentiates the given function as $y' = 2x + b$.**	**0.5**
	Finds the derivative of the given function at (1, 1) as $y' = 2 + b$.	**0.5**

Q.No	Teacher should award marks if students have done the following:	Marks
	Writes the equation of the tangent to the given curve at (1, 1) as $y - 1 = (2 + b)(x - 1)$.	**0.5**
	Assumes the point of intersection of the tangent with the x- axis as A. Finds the length of OA, where O is the origin, by substituting $y = 0$ in the above equation as $\frac{(1+b)}{(2+b)}$ units.	**0.5**
	Assumes the point of intersection of the tangent with the y- axis as B. Finds the length of OB by substituting $x = 0$ in the above equation as $-(1 + b)$ units.	**0.5**
	Mentions that ▲AOB is a right triangle. Hence, writes an expression to find the area of ▲AOB and equates it to 2 sq units as: $\frac{1}{2} \times \frac{(1+b)}{(2+b)} \times [-(1 + b)] = 2$	**1**
	Simplifies the above to get the quadratic equation: $b^2 + 6b + 9 = 0$	**1**
	Solves the above quadratic equation and finds the value of b as -3.	**0.5**
19	**i) Finds the yield(r) per day 3 years after planting the sapling as:** $r = \frac{3^3}{3} - 6(3^2) + 32(3)$ $\Rightarrow r = 51$ grams	**1**
	ii) Finds the first derivative of the given function with respect to time as: $r'(t) = t^2 - 12t + 32$	**0.5**
	Writes $r'(t) = 0$ and finds the critical points as $t = 4$ and $t = 8$.	**1**
	Writes that, since the life span of the plant is around 6 years, only $t = 4$ years is applicable here.	**0.5**

Q.No	Teacher should award marks if students have done the following:	Marks
	Finds the second derivative of the given function with respect to time as: $r''(t) = 2t - 12$	0.5
	Finds the value of $r''(t)$ **at** $t = 4$ **years as:** $r''(4) = (-4)$	0.5
	Writes that, by the second derivative test, $t = 4$ **years is the point of maxima. Hence, concludes that the maximum yield per day is obtained 4 years after planting the saplings.**	1
20	**Differentiates** x **with respect to θ as:** $\frac{dx}{d\theta} = 3 \sec\theta \tan\theta$	0.5
	Differentiates y **with respect to θ as:** $\frac{dy}{d\theta} = 3 \sec^2\theta$	0.5
	Finds $\frac{dy}{dx}$ **as:** $\frac{dy}{dx} = \frac{3\sec^2\theta}{3\sec\theta\tan\theta} = \operatorname{cosec}\theta$	0.5
	Finds the slope of the given normal line as $\frac{-1}{2}$ **. Concludes that the slope of the tangent is 2.**	1
	Equates $\frac{dy}{dx}$ **with the slope of the tangent to find the value of θ as** $\frac{\pi}{6}$ **.**	0.5
	Finds the point of contact between the tangent and curve as: $x = 3 \sec\frac{\pi}{6} = \frac{6}{\sqrt{3}}$ **or** $2\sqrt{3}$ $y = 3 \tan\frac{\pi}{6} = \frac{3}{\sqrt{3}}$ **or** $\sqrt{3}$	1
	Writes the equation of the tangent as $y - \sqrt{3} = 2(x - 2\sqrt{3})$ **or** $2x - y = 3\sqrt{3}$**.**	1

Chapter - 7
Integrals

Q: 1 **Mr. Dinesh writes the following expression on the blackboard:**

$$\int f(a)\,da = -\cos(a^2 + 1) + C$$

He asked his students to determine the function that he integrated.

Sameer says, "sin(a^2 + 1)".
Danish says, "-sin(a^2 + 1)".
Meera says, "2 a sin(a^2 + 1)".
Deepa says, "-2 a sin(a^2 + 1)".

Who is correct?

1 Sameer   2 Danish   3 Meera   4 Deepa

Q: 2 **Read the statements independently and carefully and then choose the option that correctly describes them.**

$$P = \int_a^b f(x)$$

Statement 1 : If the function $f(x)$ is well defined and continuous in the interval (a, b), then P is always positive.

Statement 2: **If P exists, then the function $f(x)$ is always continuous in the interval (a, b).**

1 Statement 1 is true but Statement 2 is false
2 Statement 1 is false but Statement 2 is true
3 Both Statement 1 and Statement 2 are true
4 Both Statement 1 and Statement 2 are false

Q: 3 **$g(x)$ + C is an integral of $h(x)$, where C is an arbitrary constant.**

$\log_e(2x + g^2(x))$ + C is an integral of which of the following?

1 $\dfrac{2h(x)g(x) + 2}{2x + h^2(x)}$ 2 $\dfrac{2g(x)h(x) + 2}{2x + g^2(x)}$ 3 $\dfrac{2g(x) + 2}{2x + g^2(x)}$ 4 $\dfrac{1}{2x + g^2(x)}$

Q: 4 **Look at the integral given below and find *f* (*x*). Show your steps.** **[1]**

$$\int f(x)\,dx = \log\left|\log x\right| + C$$

where C is an arbitrary constant.

Q: 5 **If *g* (*x*) is a polynomial function, find:** **[1]**

$$\int_{3}^{3} g(x)\,dx$$

Justify your answer.

Q: 6 **Solve:** **[2]**

$$\int \frac{6x}{x^2+2}\,dx + \int \frac{4}{x\,(x^2+2)}\,dx$$

Show your steps.

Q: 7 **Shown below is the first step of solving an integral.** **[2]**

$$\int \cot x\, g(x)\,dx = \cot x \sin x + \int \csc^2 x \sin x\,dx$$

Find *g* (*x*). Show your steps.

Q: 8 **Integrate:** **[2]**

$$\int \frac{\sin 2x\left(\frac{3}{2}\sin x - 1\right)}{e^{(\cos^2 x + \sin^3 x)}}\,dx$$

Show your steps.

Q: 9 **[2]**

$$\int \frac{1}{p^2(\sqrt{1+p^2})}\,dp$$

Lalitha integrates the above problem as shown below. She made an error in one of the steps.

Step 1: Let $p = \tan\theta$

$\Rightarrow dp = \sec^2\theta\, d\theta$

Step 2: $\int \frac{\sec^2\theta}{\tan^2\theta\,\sec\theta}\, d\theta$

Step 3 : $\Rightarrow \int \operatorname{cosec}\theta \cot\theta\, d\theta$

Step 4: $\Rightarrow$ cosec θ + C, where C is an arbitrary constant.

Step 5: $\Rightarrow \frac{\sqrt{p^2+1}}{p} + C$

In which step did she go wrong? Explain the error.

Q: 10 **[3]**

$f(x)$ is a function such that:

$$f(x) + C = \int (g(x) \times h(x))\, dx$$

where C is an arbitrary constant.

If $g'(x) = \frac{1}{x}$ and $\int h(x)\, dx = x^2$, find $f(x)$. Show your steps.

Q: 11 **Evaluate:** **[3]**

$$\int_0^1 e^x (\sec x + \sec x \tan x)\, dx$$

Show your steps.

Q: 12 **In the equation below, p, q, r and s are constants.** **[3]**

$$-9\int x\, e^{-3x} dx = e^{-3x}(\mathbf{p}x^3 + \mathbf{q}x + \mathbf{r}) + \mathbf{s}$$

Find the values of p, q, r and s. Show your steps.

Q: 13 **If *u* and *v* are two functions in *x*, integrate:** **[3]**

$$\int x^2[u(x^3)v''(x^3) - u''(x^3)v(x^3)]dx$$

Show your steps.

Q: 14 **Evaluate:** **[5]**

$$\int_2^5 \frac{x^2 + 3}{(x^2 + 2)(x^2 - 5)}\, dx$$

Show your steps.

Q: 15 **Integrate the function given below.** **[5]**

$$\int \sqrt{\sqrt{x^2 + 4} + x}\, dx$$

Show your steps.

Use the information given below to answer the questions that follow.

The derivative of the velocity function, $v(t)$, with respect to time, t, gives us the acceleration of the object. This can be written as $\frac{dv}{dt} = a(t)$ (in m/s^2).

The derivative of the displacement function, $x(t)$, with respect to time, t, gives us the velocity of the object. This can be written as $\frac{dx}{dt} = v(t)$ (in m/s).

A car is moving at a constant velocity of 15 m/s. It starts decelerating when it is about to reach its destination with acceleration, $a(t) = -\frac{t}{3}$ m/s^2 where t is the time (in seconds) for which it decelerated before coming to rest.

Q: 16 Find the velocity function of the car after it started decelerating. Show your steps. **[2]**

Q: 17 Find the time it takes the car to stop after it starts decelerating. Show your work and give a valid reason. **[1]**

Q: 18 Find the displacement function of the car and calculate its displacement from the moment it starts decelerating till it stops. Show your steps. **[2]**

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	3
2	2
3	2

Q.No	Teacher should award marks if students have done the following:	Marks
4	Differentiates log |log x | + C using the chain rule to get $f(x)$ as $\frac{1}{x.\log x}$.	1
5	Writes the value of the definite integral as 0.	0.5
	Writes that, since we are integrating the function from x = 3 to x = 3, the area under the function will be zero. (Award full marks for any other valid justification.)	0.5
6	Rewrites the integral as: $$2\int \frac{3x^2+2}{x^3+2x}\,dx$$	0.5
	Substitutes $x^3 + 2x$ as t to get: $$dx = \frac{dt}{3x^2+2}$$	0.5
	Rewrites the integral as: $$2\int \frac{1}{t}\,dt$$	0.5
	Integrates the above expression and gets 2log | t | + C as the solution where C is an arbitrary constant. Substitutes t as $x^3 + 2x$ to get 2log | $x^3 + 2x$ | + C as the solution.	0.5
7	Compares the integral with the standard form of integration by parts to conclude: $$\int g(x)\,dx = \sin x$$	1.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Uses the above step to find $g(x)$ as $\cos x$ or $\cos(2n\pi + x)$ where n is a whole number.	**0.5**
8	**Substitutes ($\cos^2 x + \sin^3 x$) as t and finds dt as follows:** $dt = \sin 2x\,(\frac{3}{2}\sin x - 1)\,dx$	**1**
	Rewrites the given integral as: $\int e^{(-t)}\,dt$	**0.5**
	Integrates the above expression to get $-e^{(-t)} + C$, where C is an arbitrary constant. **Substitutes t as $\cos^2 x + \sin^3 x$ to get:** $-e^{-(\cos^2 x + \sin^3 x)} + C$	**0.5**
9	**Writes that Lalitha made an error in step 4.**	**1**
	Writes that Lalitha has integrated the expression given in step 3 as cosec θ instead of -cosec θ.	**1**
10	**Writes that:** $f(x) + C = g(x)\int h(x)\,dx - \int[g'(x)\int h(x)\,dx]\,dx$	**1**
	It is given that $g'(x) = \frac{1}{x}$. **Integrates both sides of the above equation to get $g(x) = \log x + A$ where A is an arbitrary constant.**	**0.5**

Q.No	Teacher should award marks if students have done the following:	Marks
	Substitutes the values of *g* (*x*), *g* '(*x*) and integral of *h* (*x*) in the expression obtained in step 1 to get: $f(x) + C = x^2 (\log x + A) - \int \frac{x^2}{x}\, dx$	**0.5**
	Simplifies the above equation to get the following, where B is an arbitrary constant: $f(x) + C = x^2 \log x + Ax^2 - \frac{x^2}{2} + B$	**0.5**
	Writes that $f(x) = x^2 \log x + K x^2 + D$ where $K = A - \frac{1}{2}$ and D = B - C.	**0.5**
11	**Rewrites the integral as:** $\int_0^1 e^x \sec x \, dx + \int_0^1 e^x \sec x \tan x \, dx$	**0.5**
	Applies integration by parts on the first term to get: $[e^x \sec x]_0^1 - \int_0^1 \sec x \tan x \, e^x \, dx + \int_0^1 e^x \sec x \tan x \, dx$	**1**
	Simplifies the above expression to get: $[e^x \sec x]_0^1$	**0.5**
	Finds the value of the integral as e.sec 1° - e^0.sec 0° = e.sec 1° - 1.	**1**

Q.No	Teacher should award marks if students have done the following:	Marks
12	**Differentiates both sides of the given equation with respect to x to get:** $-9x(e^{-3x}) = (e^{-3x})(3px^2 + q) + (px^3 + qx + r)(-3e^{-3x})$	1
	Simplifies the above equation as: $-9x(e^{-3x}) = (e^{-3x})(-3px^3 + 3px^2 - 3qx + q - 3r)$	0.5
	Equates the coefficients on both sides of the equation to find the values of p, q and r as 0, 3 and 1 respectively.	1
	Writes that the constant, s cannot be uniquely determined from the given information. **(Award full marks if the problem is solved by applying integration by parts to the LHS and then equating the coefficients on both sides.)**	0.5
13	**Substitutes x^3 as t and finds dt as $3x^2\,dx$.**	0.5
	Rewrites the given expression as: $\frac{1}{3}[u(t)v''(t) - u''(t)v(t)]dt$	0.5
	Uses integration by parts for each term of the above expression as: $\frac{1}{3}[u(t)v'(t) - \int u'(t)v'(t)dt - v(t)u'(t) + \int v'(t)u'(t)dt] + C$ $\Rightarrow \frac{1}{3}[u(t)v'(t) - v(t)u'(t)] + C$ **where C is an arbitrary constant.**	1.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Substitutes *t* as x^3 and writes the final expression as: $\frac{1}{3}[u(x^3)v'(x^3) - v(x^3)u'(x^3)] + C$	**0.5**
14	**Assumes $x^2 = y$ and rewrites the given expression as follows:** $\frac{x^2+3}{(x^2+2)(x^2-5)} = \frac{y+3}{(y+2)(y-5)}$	**0.5**
	Rewrites the above expression using the partial fraction method as: $\frac{-1}{7(y+2)} + \frac{8}{7(y-5)}$	**1.5**
	Writes the integral as: $-\frac{1}{7}\int_2^5 \frac{1}{x^2+2}\,dx + \frac{8}{7}\int_2^5 \frac{1}{x^2-5}\,dx$	**0.5**
	Solves the integral as: $-\frac{1}{7}\left[\frac{1}{\sqrt{2}}\tan^{-1}\frac{x}{\sqrt{2}}\right]_2^5 + \frac{8}{7}\left[\frac{1}{2\sqrt{5}}\log\left\|\frac{x-\sqrt{5}}{x+\sqrt{5}}\right\|\right]_2^5$	**1**
	Evaluates the above integral as: $\frac{1}{7\sqrt{2}}\left(\tan^{-1}\sqrt{2} - \tan^{-1}\frac{5}{\sqrt{2}}\right) + \frac{4}{7\sqrt{5}}\left(\log\left\|\frac{5-\sqrt{5}}{5+\sqrt{5}}\right\| - \log\left\|\frac{2-\sqrt{5}}{2+\sqrt{5}}\right\|\right)$	**1.5**

Q.No	Teacher should award marks if students have done the following:	Marks
15	**Substitutes $\sqrt{x^2 + 4} + x$ as t and finds x as:** $\sqrt{x^2 + 4} = t - x$ **Squares both sides to get:** $2tx = t^2 - 4$ $x = \frac{t}{2} - \frac{2}{t}$	1.5
	Finds dx as $\left(\frac{1}{2} + \frac{2}{t^2}\right) dt$.	1
	Rewrites the integral as: $\int \sqrt{t}\left(\frac{1}{2} + \frac{2}{t^2}\right) dt$ $\Rightarrow \frac{1}{2}\int t^{\frac{1}{2}}\,dt + 2\int t^{\frac{-3}{2}}\,dt$	1
	Integrates the above expression as: $\frac{(t)^{\frac{3}{2}}}{3} - 4(t)^{-\frac{1}{2}} + C$ **where C is an arbitrary constant.**	1
	Substitutes t as $\sqrt{x^2 + 4} + x$ to get: $\frac{(\sqrt{x^2+4}+x)^{\frac{3}{2}}}{3} - \frac{4}{(\sqrt{x^2+4}+x)^{\frac{1}{2}}} + C$	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
16	**Writes the velocity function of the car (in m/s) as:** $v(t) = \int a(t)\,dt$ or $-\frac{1}{3}\int t\,dt$	0.5
	Integrates the above expression to get *v* (*t*) as follows: $v(t) = -\frac{1}{3} \times \frac{t^2}{2} + C$ **where C is an arbitrary constant.**	1
	Writes that, at *t* = 0, *v* (0) = 15 m/s. **Uses the above steps to find C as 15 and writes that:** $v(t) = -\frac{t^2}{6} + 15$	0.5
17	**Writes that the velocity of the car will be zero when the car stops. Hence, gets the equation:** $0 = 15 - \frac{t^2}{6}$	0.5
	Solves the above equation to find *t* as √90 or 3√10 seconds.	0.5
18	**Writes the displacement function of the car (in m) as:** $x(t) = \int v(t)\,dt$ or $\int\left(15 - \frac{t^2}{6}\right)dt$	0.5
	Integrates the above expression to get: $x(t) = 15t - \frac{t^3}{18} + C$ **where C is an arbitrary constant.**	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Writes that, at t = 0, the car starts decelerating until it stops and x (0) = 0. **Substitutes these values in the above expression to find C as 0 and rewrites the displacement function x (t) as:** $x(t) = 15t - \frac{t^3}{18}$	**0.5**
	Finds the displacement from the moment it starts decelerating until it stops as: $x(3\sqrt{10}) = 15 \times 3\sqrt{10} - \frac{3\sqrt{10} \times 3\sqrt{10} \times 3\sqrt{10}}{18}$ $= 45\sqrt{10} - 15\sqrt{10}$ $= 30\sqrt{10}$ **m**	**0.5**

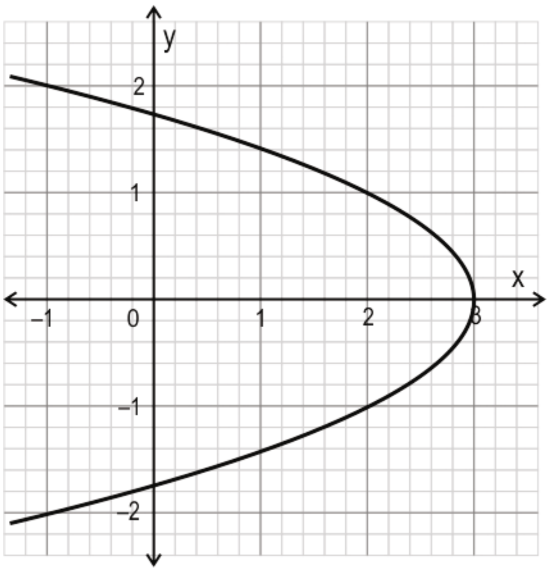
Chapter - 8
Application of Integrals

Q: 1 **If the temperature of an electric oven is increasing at the rate of *R* (*t*) °C per minute, then what does the following expression represent?**

$$\int_0^{15} R^{\prime}(t)\, dt$$

1 Rate of increase of temperature from the 0-minute mark to the 15-minute mark
2 Average increase in the temperature from the 0-minute mark to the 15-minute mark
3 Average rate of increase of temperature from the 0-minute mark to the 15-minute mark
4 Difference in the rate of increase of temperature at the 15-minute mark and the 0-minute mark

Q: 2 **Shown below is the graph of ($-x = y^2 - 3$).**



Ravi says that the area under the curve in the first quadrant can be found as:

$$\int_0^{3} \sqrt{3-x}\, dx$$

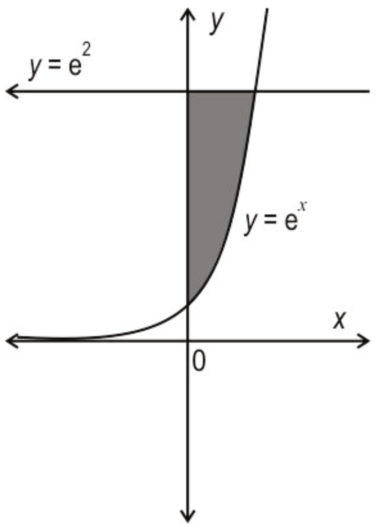
Kanika says that the area under the curve in the first quadrant can be found as:

$$\int_0^{\sqrt{3}} (3-y^2)\, dy$$

Who is correct?

1 only Ravi
2 only Kanika
3 both Ravi and Kanika
4 neither Ravi nor Kanika

Q: 3 **The shaded region shown below is bounded by the curve $y = e^x$, the y -axis and the line $y = e^2$.**



Which of the following is the area of the shaded region?

1. $\int_{1}^{e^2} (e^2 - e^x)\, dx$
2. $\int_{1}^{2} (e^2 - e^x)\, dx$
3. $\int_{1}^{e^2} e^x\, dx$
4. $\int_{0}^{2} e^x\, dx$

Q: 4 **The area under the curve $y = x^2$ between the line $x = 0$ and $x = k$ is 9 square units.**

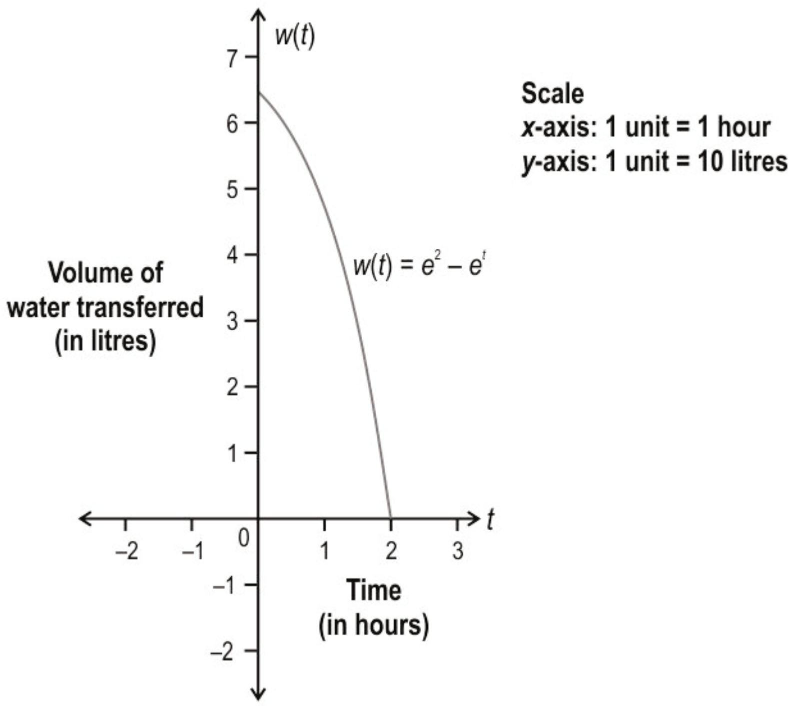
Which of the following could be the value of k?

1. 3
2. 4.5
3. 9
4. 27

Q: 5 **Water from an overhead tank is flowing down through different pipes. The volume of water (in litres) transferred at time *t* (in hours) is given by the function:**

$w(t) = e^2 - e^t$

Shown below is the graph of *w* (*t*).

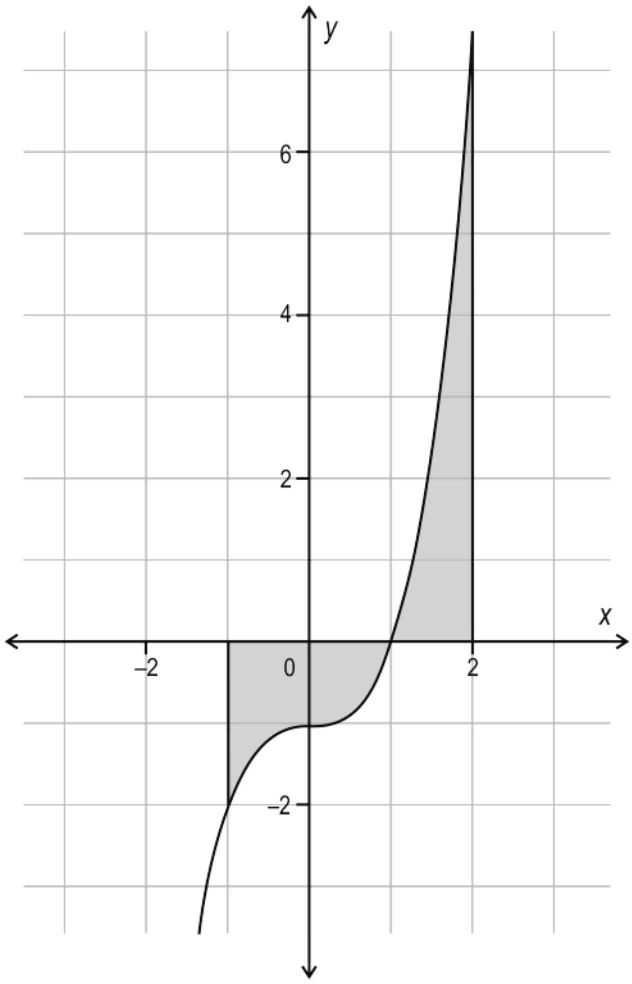


Approximately, what is the total volume of water transferred from the overhead tank in 2 hours?

***(Note: Take e*^{2} = 7.389.)**

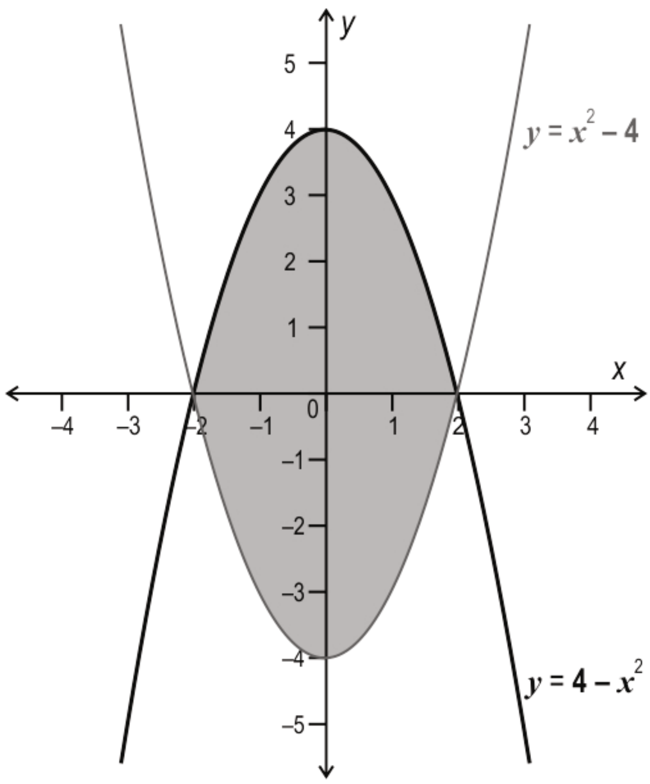
1. 6.4 litres
2. 8.4 litres
3. 64 litres
4. 84 litres

Q: 6 **Shown below is the graph of the function $y = x^3 - 1$.** **[1]**



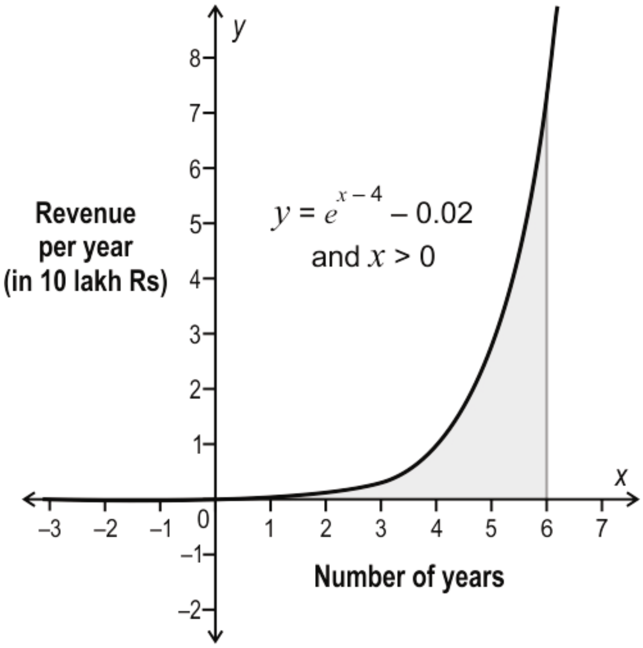
Write an expression to find the area of the shaded region by integrating with respect to the x -axis.

Q: 7 **Shown below are two curves.** **[2]**



Find the area of the shaded region. Show your steps.

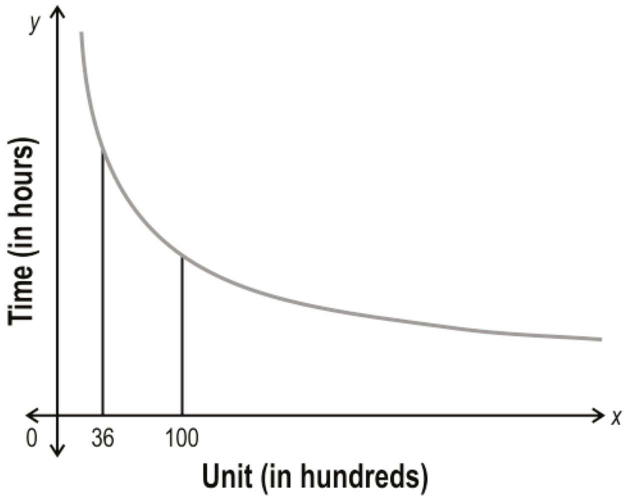
Q: 8 **A taxi company projected its revenue for 6 years using the graph shown below. The shaded region represents the total revenue that can be generated over 6 years.** **[2]**



Find the total revenue that will be generated over the last two years based on the projection. Show your steps.

(Note: Take $e^2 = 7.39$.)

Q: 9 **In economics, a learning curve is defined as a curve that shows the relation between the number of units produced and the time taken to produce them over a given period. Shown below is the learning curve of Hunar Cosmetics Private Limited.** **[2]**

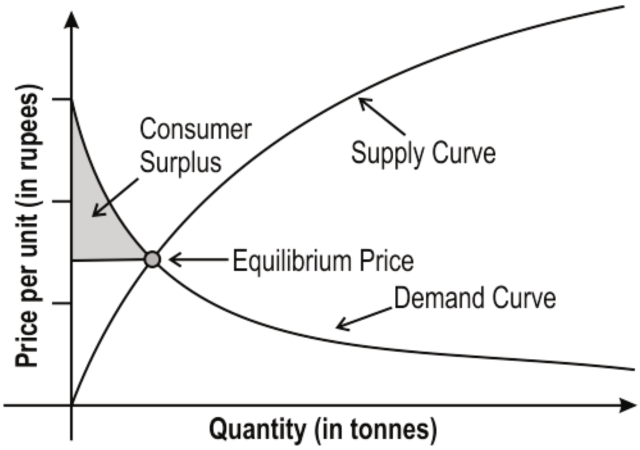


(Note: The figure is not to scale.)

The learning curve is represented by $y = 1272\, x^{\frac{-1}{2}}$.

Find the total hours required to produce the units from the 3600 th to the 10000 th unit. Show your steps.

Q: 10 **In economics, the equilibrium price is the point of intersection of the supply curve and demand curve. Consumer surplus is defined as the area covered between the demand curve and the equilibrium price line (see the shaded region in the image below).** **[3]**



For rice, assume that the demand curve is given by $\frac{180}{r+2}$ and supply curve is given by $\frac{56r}{r+1}$ where *r* is the quantity of rice (in tonnes).

Determine

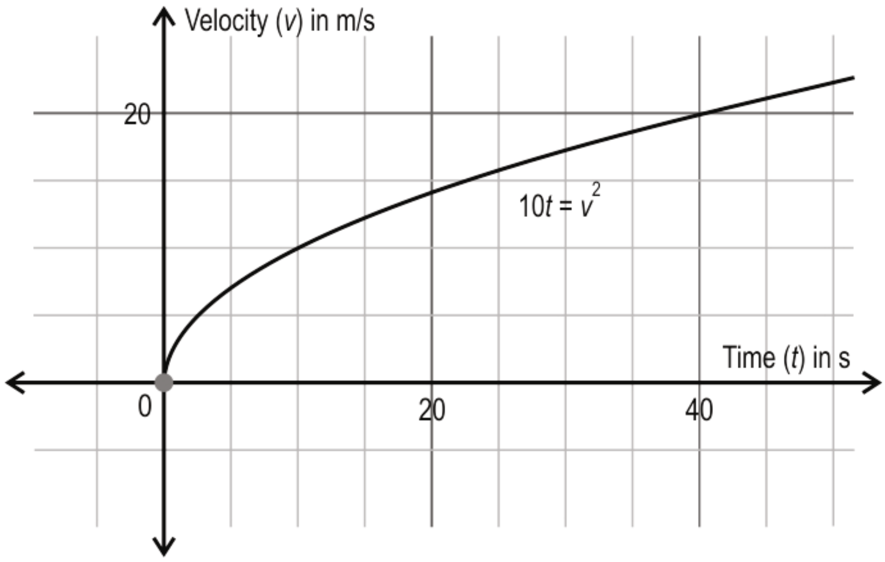
i) the equilibrium price for rice.
ii) the consumer surplus.

Show your steps.

(Note: Use ln(2) = 0.69, ln(2.25) = 0.81, ln(4.5) = 1.50)

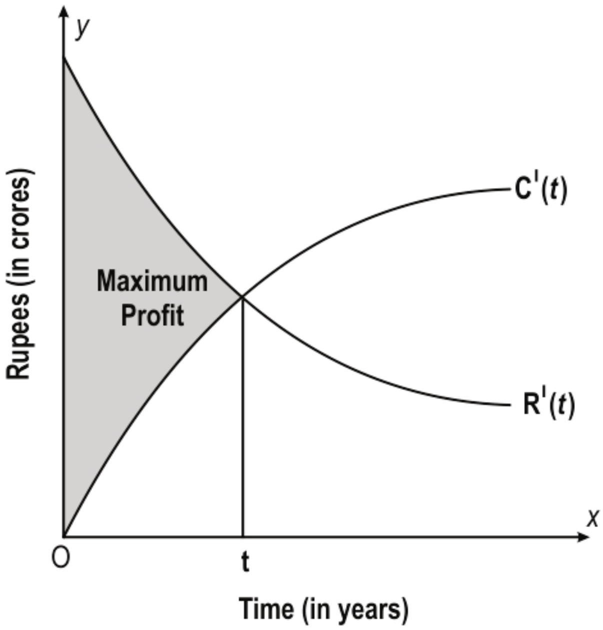
Q: 11 **The area under the velocity-time graph gives the displacement of a moving object.** **[3]**

Shown below is the velocity-time graph of a bird's flight.



Find the displacement of the bird in the first 40 seconds. Show your work.

Q: 12 **In economics, the number of years for which any company will be profitable can be found at the point of intersection of the marginal revenue (*R*') curve and the marginal cost (*C*') curve. The maximum profit a company can earn during this time is defined as the area between the *R*' curve, the *C*' curve and the *y* -axis as shown in the figure.** **[3]**



For Indian Collection Limited,

Marginal revenue function is given by $R'(t) = 10 - t^{\frac{1}{3}}$

Marginal cost function is given by $C'(t) = 2 + 3t^{\frac{1}{3}}$

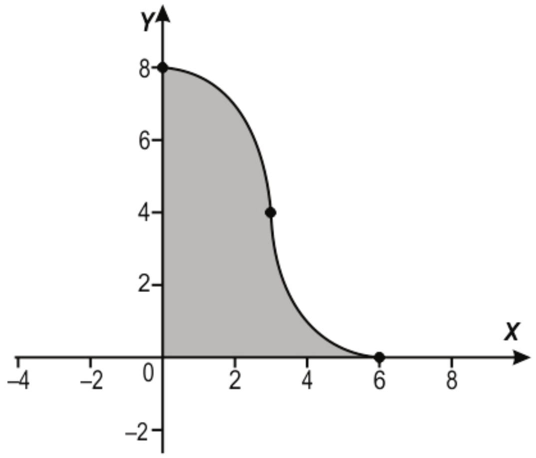
where t **is the number of years and** R' **and** C' **are in crores of rupees.**

Find:

i) the number of years for which Indian Collection Limited will be profitable.
ii) the maximum profit that Indian Collection Limited can earn during that period.

Show your steps.

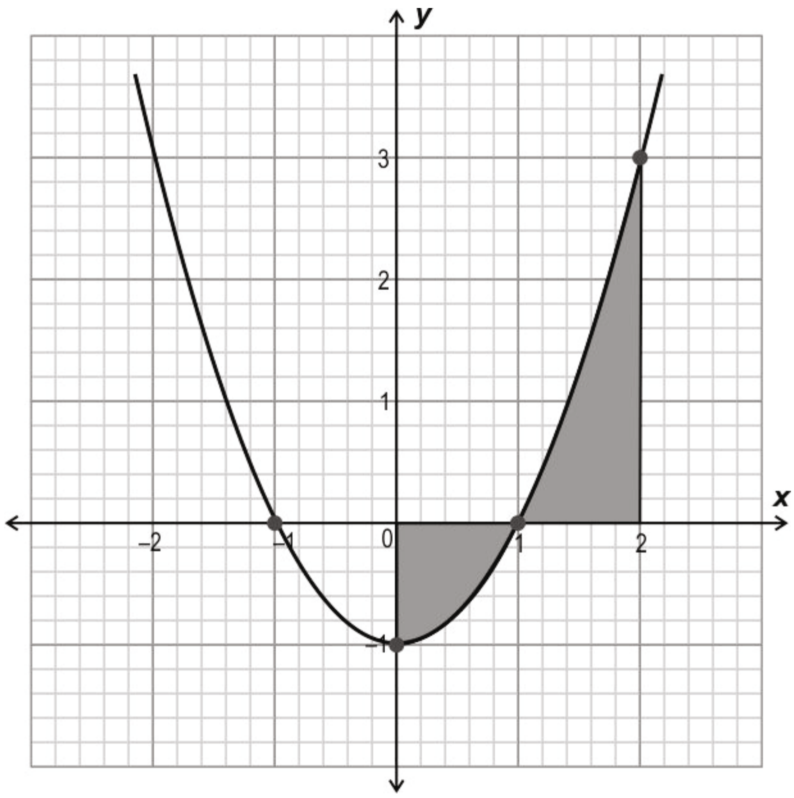
Q: 13 **A function *f* (*x*) is graphed and defined below for $x \in [0, 6]$.** **[5]**



$$f(x) = \begin{cases} 4 + 4\sqrt{1 - \dfrac{x^2}{9}} & \text{if } x \in [0, 3) \\ 4 - 4\sqrt{1 - \dfrac{(x-6)^2}{9}} & \text{if } x \in [3, 6] \end{cases}$$

Find the area of the shaded region. Show your steps.

Q: 14 **Shown below is the graph of the function $y = x^2 - 1$.** **[5]**



i) Compute the area of the shaded region. Show your work.

ii) Write an expression to find the area of the shaded region in the fourth quadrant by integrating with respect to the *y* -axis.

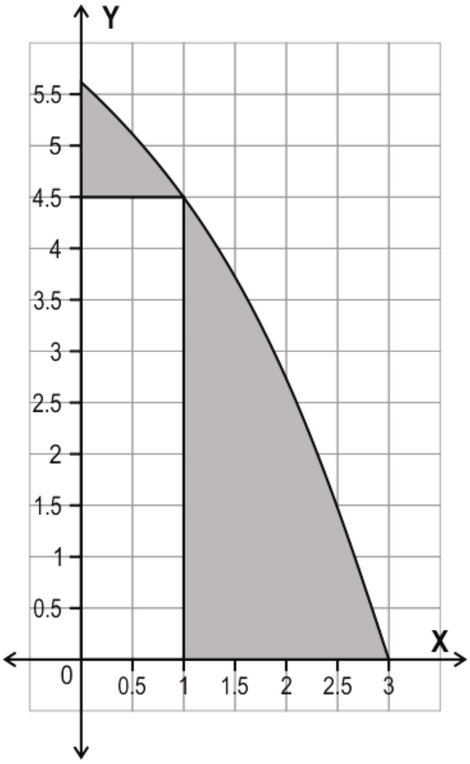
Q: 15 **i) Find the area under the curve of $f(x)$ in the first quadrant from 0 to π, where $f(x)$ is given by:** **[5]**

$$f(x) = \frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x}$$

ii) Frame an expression to find the area of sin 2 *x* in the first quadrant from 0 to π.

Show your steps and give valid reasons.

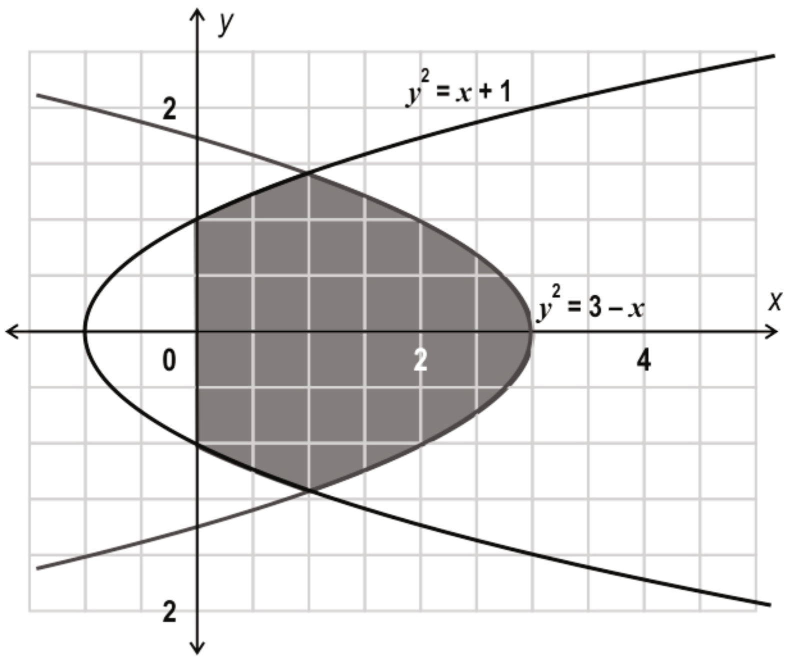
Q: 16 **Shown below is the graph of $f(x) = \frac{-3}{8}(x+1)^2 + 6$ in the first quadrant.** **[5]**



i) Find the area of the shaded region. Show your steps.

ii) In the second quadrant, if an identical unshaded rectangle is drawn under the graph of $f(x)$, would the area of the shaded area be the same as part i)? Justify your answer.

Q: 17 **Shown below are the graphs of two parabolas.** **[5]**



Find the area of the shaded region. Show your work and give your answer to 2 decimal places.

(Note: Use √2 = 1.4, √3 = 1.7 and √5 = 2.2.)

Answer the questions based on the given information.

Marginal revenue refers to the rate of change of total revenue with respect to the number of units sold at an instant and marginal cost refers to the rate of change of total cost with respect to the number of units produced at an instant.

The marginal revenue of DuPoint Limited from selling *x* units of personal protective equipment (PPE) kits in a day is given by (2 - 6 *x*) and its marginal cost of producing the same is given by (16 *x* - 1582). The fixed cost of producing PPE kits is Rs 1800 per day, where the fixed cost refers to the cost incurred at zero level of production. The company earns zero revenue at zero level of production.

(Revenue and cost are in Rupees.)

Q: 18 **Find the total cost function. Show your work.** **[1]**

Q: 19 **Find the profit function. Show your work.** **[2]**

(Note: Profit function = Total revenue function - Total cost function)

Q: 20 **Find the number of units of PPE kits that can be sold in a day to earn maximum profit. Show your work.** **[2]**

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	4
2	3
3	2
4	1
5	4

Q.No	Teacher should award marks if students have done the following:	Marks
6	**Writes the expression to find the area of the shaded region by integrating with respect to the *x* -axis as:** $\left\lvert \int_{-1}^{1} (x^3 - 1)\, dx \right\rvert + \int_{1}^{2} (x^3 - 1)\, dx$ **(Award 0.5 marks if only the correct expression with correct limits is written without the modulus symbol.)**	1
7	**Writes the expression for the area of the shaded region as:** $2\int_{-2}^{2} (4 - x^2)\, dx$	1
	Simplifies the above integral as: $2\left[4x - \frac{x^3}{3}\right]_{-2}^{2} = 21.33$ **Concludes that the area of the shaded region is 21.33 sq units.** **(Award full marks if the area in the first quadrant is found and then multiplied by 4.)**	1
8	**Writes the expression for the total revenue that will be generated over the last two years based on the projection as:** $\int_{4}^{6} (e^{x-4} - 0.02)\, dx$	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Solves the integral to get: $$\left[e^{x-4} - 0.02\,x\right]_4^6$$	0.5
	Simplifies the above expression as [e^2 - 0.02 × 6) - (e^0 - 0.02 × 4) = 7.39 - 0.12 - 1 + 0.08 = 6.35.	0.5
	Finds the total revenue that will be generated over the last two years based on the projection as 6.35 × 1000000 = Rs 6350000.	0.5
9	**Writes the expression for the total hours required to produce the units from 3600 th to 10000 th unit as:** $$\int_{3600}^{10000} 1272\,x^{-\frac{1}{2}}\,dx$$	0.5
	Solves the above integral as: $$= 2544\left[x^{\frac{1}{2}}\right]_{3600}^{10000}$$	1
	Solves the above integral to find the hours required to produce the units from the 3600 th to the 10000 th unit as 101760 hours.	0.5
10	**i) Writes the equation to calculate equilibrium price as:** $\frac{180}{r+2} = \frac{56r}{r+1}$ **=> 14 r^2 - 17 r - 45 = 0**	0.5
	Solves the equation in step 1 and finds r as 2.5 tonnes and equilibrium price as 40 rupees.	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	ii) Frames the consumer surplus as the area between the demand curve, the *y* -axis and *y* = 40 as: $$\text{Consumer Surplus} = \int_0^{2.5}\left(\frac{180}{r+2} - 40\right)dr$$ $$\text{Consumer Surplus} = \int_0^{2.5}\frac{180}{r+2}\,dr - 40 \times 2.5$$ **(Note: Award full marks for this step if the student directly writes the second equation.)**	1
	Finds the integral value as 180 × ln(2.25) - 100 = Rs 45.8.	1
11	**Writes the integral to find the displacement of the bird in the first 40 seconds as:** $$\sqrt{10}\int_0^{40}\sqrt{t}\;dt$$	1
	Integrates the above expression to get: $$\sqrt{10}\left.\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right\|_0^{40}$$	1
	Finds the displacement of the bird in the first 40 seconds as $\frac{1600}{3}$ m or 533.33 m.	1
12	**i) Writes the equation to find the number of years for which Indian Collection Limited will be profitable as:** $10 - t^{\frac{1}{3}} = 2 + 3\,t^{\frac{1}{3}}$	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Solves the above equation for *t* to find the total number of years for which Indian Collection Limited will be profitable as 8 years.	**0.5**
	ii) Writes the expression for the maximum profit that Indian Collection Limited can earn during that period as: $\int_0^8 [(10 - t^{\frac{1}{3}}) - (2 + 3t^{\frac{1}{3}})]\,dt$ $= \int_0^8 [8 - 4t^{\frac{1}{3}})]\,dt$	**0.5**
	Solves the above integral as: $= [8t - 3t^{\frac{4}{3}}]_0^8$	**1**
	Solves the above integral to find the maximum profit that Indian Collection Limited can earn during that period as 16 crore rupees.	**0.5**
13	**Writes the total area to be a sum of two integrals as:** $I = I_1 + I_2 = \int_0^3 \left(4 + 4\sqrt{1 - \frac{x^2}{9}}\right) dx + \int_3^6 \left(4 - 4\sqrt{1 - \frac{(x-6)^2}{9}}\right) dx$	**0.5**

Q.No	Teacher should award marks if students have done the following:	Marks
	Applies integration by parts and writes the first integral as: $I_1 = 4\int_0^3 dx + \frac{4}{3}\int_0^3 \sqrt{9 - x^2}\, dx$ $I_1 = 4\,[x]_0^3 + \frac{4}{3}\left[\frac{x}{2}\sqrt{9 - x^2} + \frac{9}{2}\sin^{-1}\frac{x}{3}\right]_0^3$	1
	Simplifies the integral and finds the value as: $I_1 = 4 \times (3 - 0) - \frac{4}{3}\left[0 + \frac{9\pi}{4} - 0 - 0\right] = 12 + \frac{4}{3}\left[\frac{9\pi}{4}\right]$ $I_1 = 12 + 3\pi$ sq units	1
	Applies integration by parts and writes the second integral as: $I_2 = 4\int_3^6 dx + \frac{4}{3}\int_3^6 \sqrt{9 - (x-6)^2}\, dx$ $I_2 = 4\,[x]_3^6 + \frac{4}{3}\left[\frac{(x-6)}{2}\sqrt{9 - (x-6)^2} + \frac{9}{2}\sin^{-1}\frac{(x-6)}{2}\right]_3^6$	1
	Simplifies the integral and finds the value as: $I_2 = 4 \times (6 - 3) - \frac{4}{3}\left[0 + 0 - 0 + \frac{9\pi}{4}\right] = 12 - \frac{4}{3}\left[\frac{9\pi}{4}\right]$ $I_2 = 12 - 3\pi$ sq units	1
	Adds the two integrals to find the area under the curve as: **$I = I_1 + I_2 = 24$ sq units**	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
14	**i) Considers the area of the shaded region in the first quadrant as A_1.** **Writes an expression to find A_1 as:** $A_1 = \int_1^2 (x^2 - 1)\,dx$	0.5
	Considers the area of the shaded region in the fourth quadrant as A_2. **Writes an expression to find A_2 as:** $A_2 = \left\lvert \int_0^1 (x^2 - 1)\,dx \right\rvert$ **(Award 0.5 marks if only the correct expression is written without the modulus symbol.)**	1
	Integrates and computes A_1 as: $A_1 = \left[\frac{x^2}{3} - x\right]_1^2 = \frac{4}{3}$ sq units	1
	Integrates and computes A_2 as: $A_2 = \left\lvert \left[\frac{x^2}{3} - x\right]_1^2 \right\rvert = \left\lvert \frac{-2}{3} \right\rvert = \frac{2}{3}$ sq units	1
	Finds the area of the shaded region as $A_1 + A_2 = \frac{4}{3} + \frac{2}{3} = 2$ sq units.	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	ii) Writes an expression to find the area of the shaded region in the fourth quadrant by integrating with respect to the y -axis as: $\left\| \int_{-1}^{0} \sqrt{y+1}\, dy \right\|$ **(Award 0.5 marks if only the correct expression is written without the modulus symbol.)**	1
15	**i) Writes that $f(x) = (\cos^2 x - \sin^2 x)$ or $\cos 2x$.**	0.5
	Writes that the curve $\cos 2x$ lies in the first quadrant for $x \in (0, \frac{\pi}{4}) \cup (\frac{3\pi}{4}, \pi]$ and justifies the limits. For example, draws a table as shown below: <table><tr><td>x</td><td>0</td><td>π/4</td><td>3π/4</td><td>π</td></tr><tr><td>cos 2 x</td><td>1</td><td>0</td><td>0</td><td>1</td></tr></table> **(Note: The students might include more values for x.)**	1
	Writes the expression for area as: $\int_{0}^{\frac{\pi}{4}} \cos 2x\, dx + \int_{\frac{3\pi}{4}}^{\pi} \cos 2x\, dx$	0.5
	Solves the integral to get: $\left[\frac{\sin 2x}{2}\right]_{0}^{\frac{\pi}{4}} + \left[\frac{\sin 2x}{2}\right]_{\frac{3\pi}{4}}^{\pi}$	0.5
	Finds the area as 1 sq unit.	1

Q.No	Teacher should award marks if students have done the following:	Marks
	ii) Writes that the curve sin 2 *x* lies in the first quadrant for $x \in (0, \frac{\pi}{2})$ and justifies the limits. For example, draws a table as shown below: <table><tr><td>*x*</td><td>0</td><td>π/4</td><td>π/2</td><td>3π/4</td><td>π</td></tr><tr><td>sin 2 *x*</td><td>0</td><td>1</td><td>0</td><td>-1</td><td>0</td></tr></table>**(Note: The students might include more values for *x* .)**	1
	Frames an expression for the area under the curve in the first quadrant as: $\int_0^{\frac{\pi}{2}} \sin 2x\, dx$	0.5
16	**i) Writes the expression to find the area under the curve in the first quadrant as:** $\text{Area} = \int_0^3 \left(\frac{-3}{8}(x+1)^2 + 6\right) dx$	0.5
	Writes the solution of the above integral as: $\text{Area} = \left[\frac{-x^3}{8} - \frac{3x^2}{8} + \frac{45x}{8}\right]_0^3$	1.5
	Finds the area under the curve in the first quadrant as $\frac{81}{8}$ sq units.	1
	Finds the area of the unshaded rectangular region as $1 \times \frac{9}{2} = \frac{9}{2}$ sq units.	0.5
	Finds the area of the shaded region as $\frac{81}{8} - \frac{9}{2} = \frac{45}{8}$ sq units.	0.5
	ii) Writes that the area of the shaded region in the second quadrant will not be the same as the first quadrant as the graph is not symmetrical about *x* -axis.	1

Q.No	Teacher should award marks if students have done the following:	Marks
17	**Writes the expression for the area enclosed between the parabolas as:** $$2\int_0^1 \sqrt{x+1}\,dx + 2\int_1^3 \sqrt{3-x}\,dx$$	**1.5**
	Integrates the above expression as: $$4\left[\frac{(x+1)^{\frac{3}{2}}}{3}\right]_0^1 - 4\left[\frac{(x+1)^{\frac{3}{2}}}{3}\right]_1^3$$	**1.5**
	Simplifies the above expression as $\frac{8\sqrt{2}}{3} - \frac{4}{3} + \frac{8\sqrt{2}}{3}$.	**1.5**
	Finds the area enclosed between the two parabolas as $\frac{16\sqrt{2}-4}{3}$ = 6.13 sq units.	**0.5**
18	**Integrates the given marginal cost function, 16 x - 1582, with respect to x to get the total cost function as TC(x) = 8 x^2 - 1582 x + k, where k is an arbitrary constant.**	**0.5**
	At x = 0, the total cost is equal to the fixed cost which is Rs 1800. Hence, k = 1800. Thus, the total cost function is 8 x^2 - 1582 x + 1800.	**0.5**
19	**Integrates the given marginal revenue function, 2 - 6 x, with respect to x to get the total revenue function as TR(x) = 2 x - 3 x^2 + k, where k is an arbitrary constant.**	**0.5**
	At x = 0, the total revenue is equal to Rs 0. Hence, k = 0. **Thus, the total revenue function is 2 x - 3 x^2.**	**0.5**
	Uses the total cost function found in the previous question and the above steps to find the profit function as: **2 x - 3 x^2 - (8 x^2 - 1582 x + 1800)** **= -11 x^2 + 1584 x - 1800**	**1**

Q.No	Teacher should award marks if students have done the following:	Marks
20	**Finds the first derivative of the profit function found in the previous question as $\frac{dx}{dP}$ = -22 x + 1584.**	**0.5**
	Calculates the critical point as x = 72 by equating -22 x + 1584 to 0.	**0.5**
	Finds the second derivative as: $$\frac{d^2x}{dP^2} = -22$$	**0.5**
	Concludes that the maximum profit that can be earned in one day is by selling 72 units of PPE kits.	**0.5**

Chapter - 9
Differential Equations

Q: 1 **A differential equation has an order of 3 and a degree of 2. Which of the following could this differential equation be?**

$$\frac{d^3y}{dx^3} - \left(\frac{d^2y}{dx^2}\right)^2 = 0$$

A

$$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 = 0$$

B

$$\left(\frac{d^3y}{dx^3}\right)^2 + \frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^3 = 0$$

C

$$\left(\frac{d^3y}{dx^3}\right)^2 + \tan\left(\frac{d^2y}{dx^2}\right) = 0$$

D

1 A

2 B

3 C

4 D

Q: 2 **Look at the differential equation given below. What is its degree?**

$$y^3 = e^{4\frac{d^2y}{dx^2}}$$

1 1

2 2

3 3

4 4

Q: 3 **Shown below is a differential equation.**

$\sin x \frac{dy}{dx} + y \cos x = \tan x$

Which of the following is the integrating factor for the above differential equation ?

$e^{\int \sin(x)dx}$

Expression 1

$e^{\int \cos(x)dx}$

Expression 2

$e^{\int \cot(x)dx}$

Expression 3

1 Expression 1

2 Expression 2

3 Expression 3

4 (the equation has no integrating factor as it is not a linear differential equation)

Q: 4 **Which of the following differential equations will give $y = \sin x + \cos x + C$ as the general solution where C is an arbitrary constant?**

$$\frac{d^2y}{dx^2} = \cos x + \sin x \qquad \frac{d^2y}{dx^2} = \cos x - \sin x \qquad \frac{d^2y}{dx^2} = -\cos x - \sin x$$

Equation 1 Equation 2 Equation 3

1. Equation 1
2. Equation 2
3. Equation 3
4. (cannot say without knowing the value of C)

Q: 5 **Three friends - Bulbul, Ipsita and Sagarika were asked to find a particular solution of the following differential equation.**

$$\frac{d^2y}{dx^2} + y = 0$$

Shown below are their solutions.

Bulbul: $y = \sin x$
Ipsita: $y = \cos x$
Sagarika: $y = \sin x + \cos x$

Whose answer is correct?

1. only Bulbul
2. only Sagarika
3. only Bulbul and Ipsita
4. All - Bulbul, Ipsita and Sagarika

Q: 6 **Read the statements carefully and choose the option that correctly describes them.**

Statement 1 : The differential equation that represents the family of straight lines with slope 7 is given by $y = 7x + c$, where c is an arbitrary constant.

Statement 2 **: $\frac{dy}{dx} = 7$ is a differential equation of order 0 as it contains no arbitrary constant.**

1. Statement 1 is true but Statement 2 is false
2. Statement 1 is false but Statement 2 is true
3. Both Statement 1 and Statement 2 are true
4. Both Statement 1 and Statement 2 are false

Q: 7 What is the order of the differential equation whose solution is given below? [1]

y = (R - S)cosec (x + T) - Ue$^{2x^{-V}}$, where R, S, T, U and V are some constants.

Show your steps and give a valid reason.

Q: 8 Three students were discussing the nature of the differential equation given below. [2]

$x + y\frac{dy}{dx} = b$, where b is a constant.

Samreen said, "This equation represents a family of straight lines".

Jamal said, "This equation represents a family of circles".

Kavya said, "This equation represents a family of ellipses."

Who is correct? Justify your answer.

Q: 9 The first derivative of a function is given below: [2]

$$\frac{-1}{x^2(1+x^2)}$$

Find the function. Show your steps.

Q: 10 The rate of decay of a radioactive isotope is given by $\frac{dN}{dt}$ = -kN, where k is a positive constant and N is the quantity (in grams) of the radioactive material available at time t (in days). k is equal to $\frac{0.693}{T}$ where T is the half-life (in days) of the isotope. [3]

If the initial quantity of the radioactive material is 45 g and its half-life is 3 days, find the quantity of the radioactive material left after 10 days. Show your steps.

(Note: Take the value of e $^{-2.31}$ as 0.099.)

Q: 11 In a research experiment, the population of fruit flies grows at the rate of 20% every day. [3]

i) Frame a differential equation depicting the above experiment.
ii) Find the time taken for the population to triple.

Show your steps.

Q: 12 ***y* = *f* (*x*) is a curve such that, at every point on the curve, the slope is half the product of the coordinates of that point.** **[3]**

i) Frame a differential equation that represents the above curve.
ii) Find the general solution of the differential equation obtained in part i). Show your work.

Q: 13 **Find the general solution of the given differential equation. Show your steps.** **[3]**

$$\frac{dy}{dx} = ye^{\ln\frac{1}{x}} + 1$$

Q: 14 **Ankita bought a car for Rs *y*. The price of the car, P(*t*) after *t* years depreciates as per the equation $\frac{dP}{dt}$ = - *a* (T - *t*) where T is the total life of the car (in years) and *a* is an arbitrary constant.** **[3]**

Find the value of the car when the car has been used for T years, P(T), in terms of *y*. Show your steps.

Q: 15 **Find the general solution of the following differential equation.** **[5]**

$$\sin x \frac{dy}{dx} = \left(y + \sqrt{\sin^2 x - y^2}\right)\cos x$$

Show your steps.

Q: 16 **Show that the given differential equation is homogeneous and solve it.** **[5]**

$\frac{dy}{dx} = \frac{y}{x} + \cos\frac{y}{x}$

Show your steps.

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	3
2	1
3	3
4	3
5	4
6	4

Q.No	Teacher should award marks if students have done the following:	Marks
7	**Rewrites the given equation in terms of arbitrary constants as:** $y = C_1 \text{cosec}(x + C_2) - C_3 e^{2x}$ **where,** $C_1 = (R - S)$, $C_2 = T$ **and** $C_3 = Ue^{-V}$	0.5
	Writes that since the order of a differential equation is same as the number of arbitrary constants, the order of the given differential equation is 3.	0.5
8	**Writes that Joseph is correct.**	0.5
	Rewrites the given differential equation as: $y\,dy = (b - x)\,dx$	0.5
	Integrates both sides of the above equation to get: $\frac{y^2}{2} = bx - \frac{x^2}{2} + K$ $\Rightarrow \frac{x^2}{2} + \frac{y^2}{2} - bx = K$ **where K is an arbitrary constant.** **Concludes that the above equation represents a family of circles.**	1
9	**Writes the derivative as:** $\frac{dy}{dx} = \frac{1}{1+x^2} - \frac{1}{x^2}$	0.5
	Rewrites both sides of the equation as: $\int dy = \int \frac{1}{(1+x^2)}dx - \int \frac{1}{x^2}dx$	0.5
	Integrates the above equation to get the function as: $y = \tan^{-1}(x) + \frac{1}{x} + C$	1

Q.No	Teacher should award marks if students have done the following:	Marks
10	Solves the differential equation: $\int \frac{dN}{N} = -k \int dt$ and writes N = $Ce^{-k\,t}$ as the solution where C is an arbitrary constant.	1
	Writes that, at t = 0, N = 45 g and hence finds C as 45.	0.5
	Calculates k as $\frac{0.693}{3}$ = 0.231	0.5
	Calculates the quantity of the radioactive material left after 10 days as: N = $45e^{-0.231 \times 10}$ = 45 × 0.099 = 4.45 g	1
11	i) Frames the differential equation depicting the given experiment as: $\frac{dP}{dt} = 0.2P$ => $\frac{dP}{P} = 0.2\,dt$ where P is the population at time t (in days).	0.5
	Integrates the above equation to get: $\log_e P = 0.2\,t + C$, where C is an arbitrary constant.	0.5
	Assumes that, at t = 0, P = P_0 and gets C = $\log_e P_0$.	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Uses steps 2 and 3 and writes: $log_e P = 0.2t + log_e P_0$ $\Rightarrow log_e \frac{P}{P_0} = 0.2t$ **(Award full marks if the equation is derived in exponential form, P = $P_0 e^{(0.2t)}$.)**	0.5
	Finds the time taken for the population to triple as: $log_e \frac{3P_0}{P_0} = 0.2t$ $\Rightarrow t = (5log_e 3)$ days	1
12	**i) Frames the differential equation representing the given curve as:** $\frac{dy}{dx} = \frac{1}{2}(xy)$	1
	ii) Separates the variables of the differential equation obtained in step 1 as: $\frac{2}{y}dy = x\,dx$	0.5
	Integrates the equation obtained in step 2 to get: $2logy = \frac{x^2}{2} + c$, where c is an arbitrary constant **Simplifies the above equation to get the general solution as:** $y = Ce^{(\frac{x^2}{4})}$, where $C = e^{(\frac{c}{2})}$ is an arbitrary constant. **(Award full marks if the final answer is written as 2log $y = x^2/2 + c$.)**	1.5

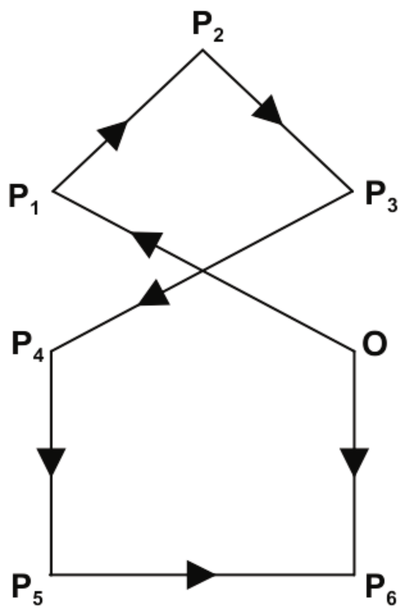
Q.No	Teacher should award marks if students have done the following:	Marks
13	**Simplifies and writes the linear differential equation as $\frac{dy}{dx} - \frac{y}{x} = 1$.**	0.5
	Finds the integrating factor as: $e^{-1\int \frac{1}{x}dx} = e^{\ln x^{-1}}$ *or* $\frac{1}{x}$	1
	Writes the solution as: $y \times \frac{1}{x} = \int 1 \times \frac{1}{x}\, dx$	1
	Simplifies the solution as: $\frac{y}{x} = \ln x + C$ **(Award full marks if any other form of this equation is written.)**	0.5
14	**Rewrites the given differential equation as:** $\int dP = -a\int (T-t)\, dt$	0.5
	Integrates the above equation as: $P(t) = \frac{at^2}{2} - atT + C$ **where C is an arbitrary constant.**	1
	Writes that, at $t = 0$, P(t) = y so C = y.	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Finds the value of the car as: $$P(T) = \frac{aT^2}{2} - aT^2 + y = y - \frac{aT^2}{2}$$	1
15	**Substitutes sin *x* as *t* and finds $\frac{dx}{dt} = \frac{1}{\cos x}$.**	0.5
	Writes that from the given equation and $\frac{dx}{dt} = \frac{1}{\cos x}$ we can find: $$\frac{dy}{dt} = \frac{y + \sqrt{t^2 - y^2}}{t}$$ **where *t* = sin *x*.**	1.5
	Substitutes *y* = *vt* and gets $\frac{dy}{dt} = v + t\frac{dv}{dt}$.	0.5
	Rewrites the equation as: $v + t\frac{dv}{dt} = v + \sqrt{(1 - v^2)}$ **where** $v = \frac{y}{t}$ **and reduces it to:** $$\frac{dv}{\sqrt{1 - v^2}} = \frac{dt}{t}$$	1
	Solves the differential equation as: $\sin^{-1}(v) = \ln\lvert t\rvert + C$ **where C is an arbitrary constant.**	0.5
	Substitutes the value of *v* as $\frac{y}{t}$ and gets the solution as: $\sin^{-1}\left(\frac{y}{t}\right) = \ln\lvert t\rvert + C$	0.5
	Substitutes the value of *t* as sin *x* and gets the final solution as: $\sin^{-1}\left(\frac{y}{\sin x}\right) = \ln\lvert\sin x\rvert + C$	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
16	Takes $F(x, y) = \frac{y}{x} + \cos\frac{y}{x}$ and finds $F(\lambda x, \lambda y)$ as: $F(\lambda x, \lambda y) = \frac{\lambda y}{\lambda x} + \cos\frac{\lambda y}{\lambda x}$ $\Rightarrow F(\lambda x, \lambda y) = \lambda^0 F(x, y)$ Hence, concludes that the given differential equation is homogeneous.	1
	Takes $y = vx$ and differentiates it to get $\frac{dy}{dx} = v + x\frac{dv}{dx}$.	1
	Rewrites the given differential equation using the above step as: $\sec v\, dv = \frac{1}{x} dx$	1
	Integrates the above equation as: $\log(\sec v + \tan v) + \log C = \log x$ where, C is an arbitrary constant.	1
	Simplifies the above equation as: $C(\sec v + \tan v) = x$	0.5
	Replaces $y = vx$ in the above equation to find the general solution of the given differential equation as: $x = C\left[\sec\frac{y}{x} + \tan\frac{y}{x}\right]$	0.5

Chapter - 10
Vector Algebra

Q: 1 **Observe the vector diagram given below.**



Which of the following is/are equal to

$\overrightarrow{OP_1} + \overrightarrow{P_1P_2} + \overrightarrow{P_2P_3} + \overrightarrow{P_3P_4} + \overrightarrow{P_4P_5} + \overrightarrow{P_5P_6}$?

i) $\overrightarrow{P_6O}$

ii) $\overrightarrow{OP_6}$

iii) $-\overrightarrow{OP_6}$

iv) $-\overrightarrow{P_6O}$

1 only i) **2** only ii) **3** only i) and iii) **4** only ii) and iv)

Q: 2

$\vec{u} = \hat{i}$, $\vec{v} = \hat{j}$ and $\vec{w} = \hat{k}$ are unit vectors. Which of the following is the angle between $(\vec{v} \times \vec{u})$ and $\vec{w}$?

1 0°

2 90°

3 180°

4 (cannot be found without knowing the angle between the vectors *v* and *u*)

Q: 3 Under which of the following conditions will the magnitude of the cross product of two vectors, $\vec{a}$ and $\vec{b}$, attain its maximum value?

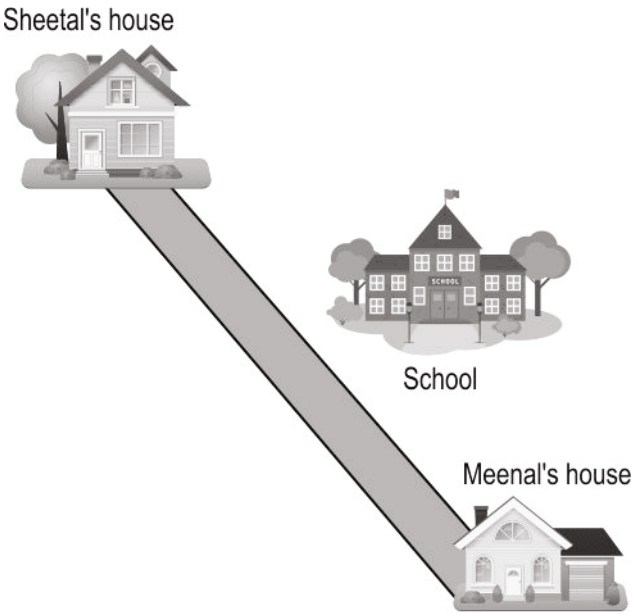
1 When $\vec{a}$ is parallel to $\vec{b}$

2 When $\vec{a}$ is perpendicular to $\vec{b}$

3 When $\vec{a}$ is equal to and parallel to $\vec{b}$

4 When the magnitudes of $\vec{a}$ and $\vec{b}$ are equal but they are in opposite direction

Q: 4 **Sheetal and Meenal walk to school from two ends of a street in a straight line path.**



(Note: The figure is not to scale.)

At a certain time, Sheetal's velocity vector is $2\hat{i} - 3\hat{j}$ m/s.

Which of the following could be Meenal's velocity vector (in m/s) at that time?

1 $4\hat{i} - 6\hat{j}$

2 $\hat{i} + \frac{3}{2}\hat{j}$

3 $-\hat{i} + \frac{3}{2}\hat{j}$

4 (cannot say without knowing Meenal's velocity)

Q: 5 The following question was given in a class test. **[1]**

"If $\vec{p} = 5\hat{i}$, $\vec{q} = -2\hat{j}$ and $\vec{r} = 3\hat{k}$, then find $(\vec{p} \times \vec{q}) + (\vec{q} \times \vec{r}) + (\vec{r} \times \vec{p})$."

Aditi wrote the answer as 0.

Karn wrote the answer as $-10\hat{i} - 6\hat{j} + 15\hat{k}$.

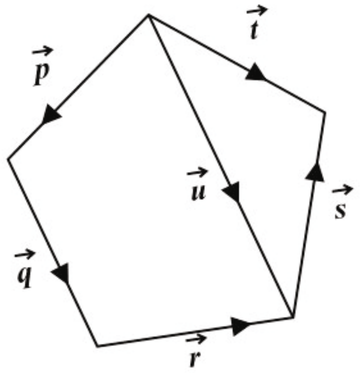
Manisha wrote the answer as $-6\hat{i} + 15\hat{j} - 10\hat{k}$.

Jamal wrote the answer as $6\hat{i} - 15\hat{j} + 10\hat{k}$.

Who was right? Justify your answer.

Q: 6 **[1]**

Two vertices of a regular pentagon are joined by a vector u (as shown below), followed by two statements. Identify if the statements are true or false. Give valid reasons for your answer.



i) $\vec{p} = \vec{q} = \vec{r} = \vec{s} = \vec{t}$

ii) There are three coinitial vectors.

Q: 7 State whether the following statement is true or false. If true, give a reason. If false, give an example. **[1]**

Two collinear vectors are always equal.

Q: 8 **[1]**

Vectors $\overrightarrow{OA} = 2\hat{i} + 6\hat{k}$ and $\overrightarrow{OB} = 4\hat{i} - 2\hat{j} + 4\hat{k}$ are two sides of a $\triangle OAB$, where O is the origin. Find the length of the median $\overrightarrow{OC}$.

Show your work with a valid reason.

Q: 9 A parallelogram, ABCD, is constructed such that its adjacent sides, AB and AD, are $3\vec{a} - 5\vec{b}$ and $\vec{a} - 2\vec{b}$ respectively. $|\vec{a}| = \sqrt{2}$, $|\vec{b}| = \sqrt{8}$ and the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$. **[2]**

Find the length of the diagonal BD. Show your steps.

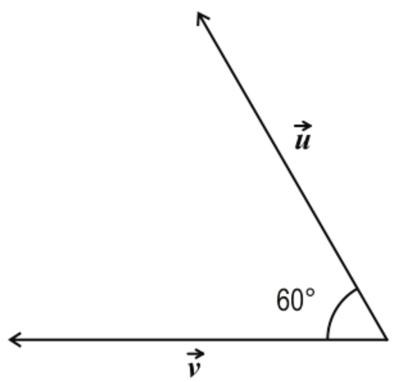
Q: 10 Two vectors are orthogonal if they are perpendicular to each other. **[2]**

Determine whether $\vec{u} = \hat{i} + 3\hat{j} + \hat{k}$ and $\vec{v} = 5\hat{i} - 2\hat{j} + \hat{k}$ are orthogonal vectors. Show your steps and give valid reasons.

Q: 11 If $\vec{u} = (t-1)\hat{i} + 2\hat{j} + (t-3)\hat{k}$ is rotated about the origin in a clockwise direction to obtain $\vec{v} = \hat{i} + (t-1)\hat{j} + 2\hat{k}$, where t is a real number, find the possible values of t. **[2]**

Show your steps and give valid reasons.

Q: 12 In the figure below, the magnitudes of vectors *u* and *v* are 5 and 3 respectively. **[2]**



(Note: The figure is not to scale.)

Using the dot product, find the magnitude of the vector (*u* + *v*). Show your steps.

Q: 13 The projection of vector $\vec{a} = 3\hat{i} + q\hat{j} - \hat{k}$ on vector $\vec{b} = \hat{i} + \sqrt{2}\hat{j} + p\hat{k}$ is 1, where p and q are natural numbers. **[2]**

If $|\vec{b}| = \sqrt{12}$, find:

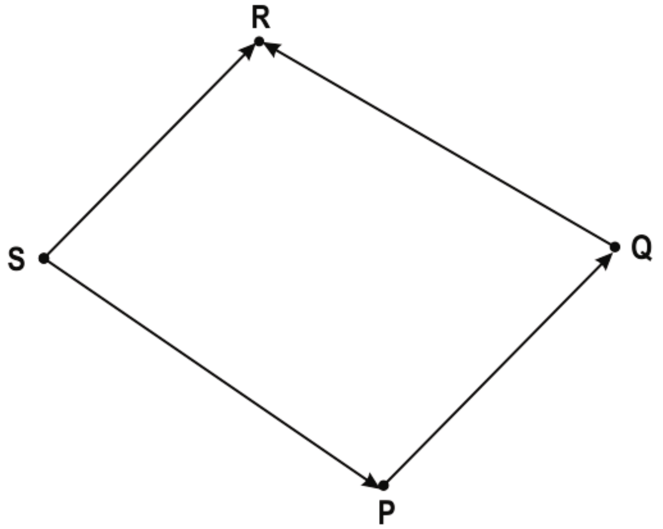
i) p

ii) q

Show your steps.

Q: 14 Find the angles made by the vector, $2\hat{i} - 3\hat{j} + \hat{k}$, with the coordinate axes. Show your steps. **[2]**

Q: 15 In the figure below, point S is at the origin. The position vectors of points P, Q and R are $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively. **[3]**



Prove that $\overrightarrow{PQ} \times \overrightarrow{RS} + \overrightarrow{QR} \times \overrightarrow{PS} + \overrightarrow{RP} \times \overrightarrow{QS} = 2(\overrightarrow{SR} \times \overrightarrow{SQ} + \overrightarrow{SP} \times \overrightarrow{SR} + \overrightarrow{SQ} \times \overrightarrow{SP})$.

Q: 16 Using vector operations, find the area of a parallelogram RSTU with vertices R(-1, 2, 3), S(-2, 5, -1) and U(0, 0, 3). Show your steps. **[3]**

Q: 17 A vector $\vec{v}$ has a magnitude of 2, makes an angle of $\frac{\pi}{6}$ with $\hat{i}$, an angle of θ with $\hat{j}$ and an angle of $\frac{\pi}{3}$ with $\hat{k}$. Find: **[3]**

i) θ, where $0 \leq \theta \leq 90°$

ii) $\vec{v}$ in its component form

Show your steps.

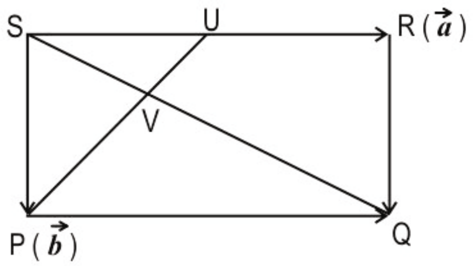
Q: 18 **PQRS is a rhombus. Points X and Y are the midpoints of the sides SP and PQ respectively.** **[3]**

Draw a vector diagram and prove that:

$$\overrightarrow{RX} + \overrightarrow{RY} = \frac{3}{2}\overrightarrow{RP}$$

Q: 19 **[5]**

In the rectangle PQRS below, S is at the origin and the position vectors of the points P and R are $\vec{b}$ and $\vec{a}$ respectively.



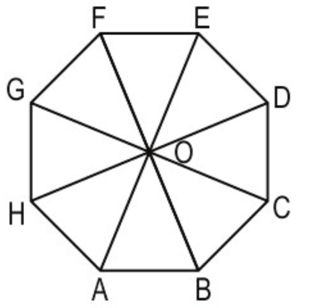
(Note: The figure is not to scale.)

i) Point U bisects SR. Using the vector method, prove that V divides PU and QS in the same ratio.

ii) Find the ratio in which point V divides QS.

Show your steps and give valid reasons.

Q: 20 **Shown below is a regular octagon, ABCDEFGH, with centre at O.** **[5]**



Show that $\overrightarrow{AE} + \overrightarrow{FB} + \overrightarrow{CG} + \overrightarrow{HD} = 2(\overrightarrow{AD} - \overrightarrow{BC})$.

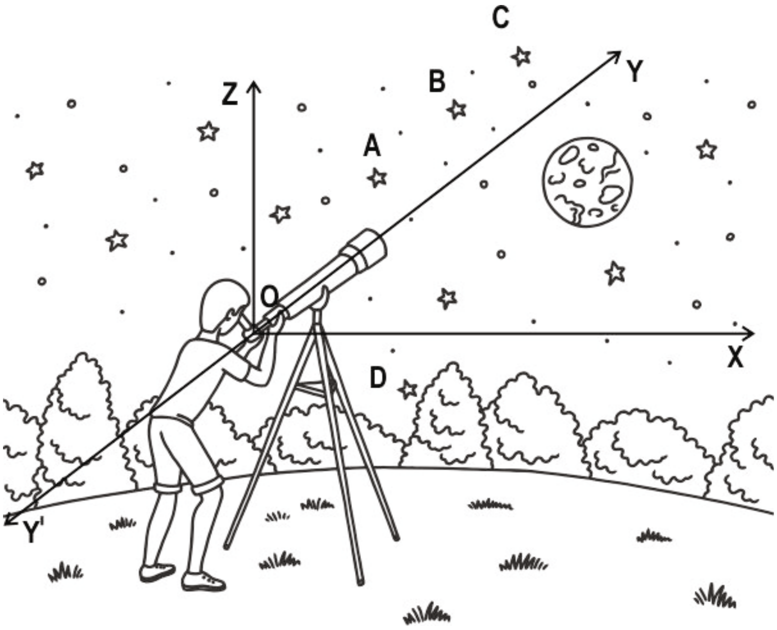
Q: 21 **PQR is a right-angled isosceles triangle with ∠Q = 90°. O is a point on the hypotenuse of ▲PQR and S is a fixed point outside the triangle such that:** **[5]**

$$\overrightarrow{OS} = \overrightarrow{QO} + \overrightarrow{OR} + \overrightarrow{OP}$$

Draw a vector diagram and prove that PQRS is a square.

Read the given information and answer the questions that follow.

On a certain night, Ziad was observing stars using his telescope.



(Note: Image is purely for representation purposes.)

Considering the eyepiece of the telescope as the origin, the position vectors of some of the stars are given below:

A $(2\hat{i} + 3\hat{j} + 4\hat{k})$

B $(5\hat{i} + 6\hat{j} + 8\hat{k})$

C $(8\hat{i} + 9\hat{j} + 12\hat{k})$

D $(2\hat{i} + 3\hat{j} - 6\hat{k})$

Q: 22 **Show that the stars with position vectors A, B and C are collinear.** **[2]**

Q: 23 Express the vector $\overrightarrow{AD}$ in terms of its components and find the distance between the stars A and D. **[1]**

(*Note: 1 unit = 1 light year.*)

Q: 24 **A star at a certain position is inclined at angles of 45° and 60° with the positive direction of the *x* -axis and *y* -axis respectively.** **[2]**

If the star is 2 light years away from O, find its position vector.

(Note: 1 unit = 1 light year.)

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	4
2	3
3	2
4	3

Q.No	Teacher should award marks if students have done the following:	Marks
5	**Writes that Manisha was right.**	0.5
	Justifies the above answer as follows: $(\vec{p}\times\vec{q})+(\vec{q}\times\vec{r})+(\vec{r}\times\vec{p})$ $= (5\hat{i}\times -2\hat{j})+(-2\hat{j}\times 3\hat{k})+(3\hat{k}\times 5\hat{i})$ $= -10(\hat{i}\times\hat{j})-6(\hat{j}\times\hat{k})+15(\hat{k}\times\hat{i})$ $= -6\hat{i}+15\hat{j}-10\hat{k}$	0.5
6	**i) Writes False(F). Gives the reason that all the five vectors mentioned in i) have different directions.**	0.5
	ii) Writes True(T). Gives the reason that the following three vectors have the same initial point. $\vec{p}$, $\vec{u}$ and $\vec{t}$	0.5
7	**Writes false.**	0.5
	Gives an example: $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{b} = 6\hat{i} + 9\hat{j} + 12\hat{k}$ are collinear vectors that are not equal.	0.5
8	**Writes that, since $\overrightarrow{OC}$ is the median, C is the midpoint of $\overrightarrow{AB}$. Hence, writes $\overrightarrow{OC} = \frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OB}) = \frac{1}{2}(6\hat{i} - 2\hat{j} + 10\hat{k}) = 3\hat{i} - \hat{j} + 5\hat{k}$.**	0.5

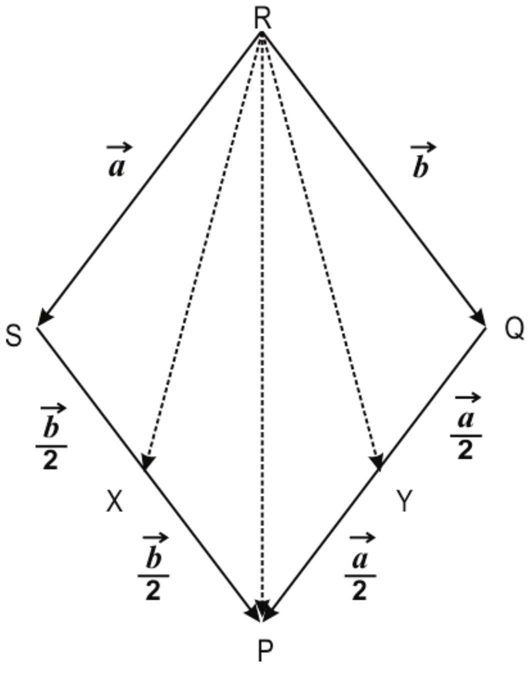
Q.No	Teacher should award marks if students have done the following:	Marks
	Finds the length of the median as $\lvert\overrightarrow{OC}\rvert = \sqrt{(9 + 1 + 25)} = \sqrt{35}$ units.	0.5
9	**Writes that the length of the diagonal, BD is represented by:** $\lvert\overrightarrow{BD}\rvert = \lvert 3\vec{a} - 5\vec{b} - (\vec{a} - 2\vec{b})\rvert = \lvert 2\vec{a} - 3\vec{b}\rvert$	0.5
	Finds $\lvert\vec{BD}\rvert^2$ as follows: $(2\vec{a} - 3\vec{b}).(2\vec{a} - 3\vec{b})$ $= 4\lvert\vec{a}\rvert^2 - 12(\vec{a}.\vec{b}) + 9\lvert\vec{b}\rvert^2$ $= 8 - 12(\sqrt{2} \times \sqrt{8} \times \cos\frac{\pi}{3}) + 72$ $= 80 - 24$ $= 56$	1
	Finds the length of the diagonal, BD as √56 units.	0.5
10	**Finds the dot product of the given two vectors as:** $\vec{u} . \vec{v} = (1)(5) + (3)(-2) + (1)(1) = 0$	0.5
	Finds the angle between $\vec{u}$ and $\vec{v}$ as follows: $\vec{u} . \vec{v} = 0$ $\Rightarrow \lvert\vec{u}\rvert\,\lvert\vec{v}\rvert \cos\theta = 0$ As $\lvert\vec{u}\rvert \neq 0$ and $\lvert\vec{v}\rvert \neq 0$, $\cos\theta = 0$. $\Rightarrow \theta = 90°$	1
	Concludes that $\vec{u}$ and $\vec{v}$ are orthogonal vectors.	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
11	**Writes that the magnitudes of the two vectors will be the same even after rotation.** **Hence, writes that:** $\lvert\vec{u}\rvert = \lvert\vec{v}\rvert \Rightarrow \lvert\vec{u}\rvert^2 = \lvert\vec{v}\rvert^2$	0.5
	Uses the above step and writes: $(t - 1)^2 + 2^2 + (t - 3)^2 = 1^2 + (t - 1)^2 + 2^2$	0.5
	Simplifies the above equation as: $(t - 3) = \pm 1$	0.5
	Simplifies the above equation to find the values of *t* as 2 and 4.	0.5
12	Finds $\lvert\vec{u}+\vec{v}\rvert^2$ as follows : $= (\vec{u}+\vec{v}).(\vec{u}+\vec{v})$ $= \lvert\vec{u}\rvert^2 + 2(\vec{u}.\vec{v}) + \lvert\vec{v}\rvert^2$ $= 25 + 2(5 \times 3 \times \cos 60^\circ) + 9$ $= 49$	1.5
	Finds the magnitude of the vector $(\vec{u}+\vec{v})$ as $\sqrt{49}$ or 7 units.	0.5
13	**i) Uses magnitude of vector *b* to find *p* as:** $\lvert\vec{b}\rvert = \sqrt{1 + (\sqrt{2})^2 + p^2} = \sqrt{12}$ $\Rightarrow p^2 = 9$ $\Rightarrow p = 3$ as $p > 0$ **(Award only 0.5 marks if the formula to find the magnitude of the vector is correctly written.)**	1

Q.No	Teacher should award marks if students have done the following:	Marks
	ii) Uses the value of *p* found in step 1 and the formula for the projection of a vector to find *q* as: $\frac{\vec{a}\cdot\vec{b}}{\lvert\vec{b}\rvert} = \frac{a_1b_1 + a_2b_2 + a_3b_3}{\lvert\vec{b}\rvert}$ $\Rightarrow \frac{(3)(1)+(q)(\sqrt{2})+(-1)(3)}{\sqrt{12}} = 1$ $\Rightarrow q = \frac{\sqrt{12}}{\sqrt{2}} = \sqrt{6}$ **(Award only 0.5 marks if the formula to find the projection of a vector on another vector is correctly written.)**	1
14	**Finds the magnitude of the vector as √(4 + 9 + 1) or √14 units.**	0.5
	Finds the direction cosines of the vector as $\frac{2}{\sqrt{14}}, \frac{-3}{\sqrt{14}}, \frac{1}{\sqrt{14}}$.	0.5
	Writes that the given vector makes angles: **$\cos^{-1}\frac{2}{\sqrt{14}}$ with *x* -axis** **$\cos^{-1}\frac{-3}{\sqrt{14}}$ with *y* -axis** **$\cos^{-1}\frac{1}{\sqrt{14}}$ with *z* -axis**	1
15	Writes that $\overrightarrow{PQ} \times \overrightarrow{RS} = (\vec{b} - \vec{a}) \times (-\vec{c})$.	0.5
	Writes that $\overrightarrow{QR} \times \overrightarrow{PS} = (\vec{c} - \vec{b}) \times (-\vec{a})$.	0.5
	Writes that $\overrightarrow{RP} \times \overrightarrow{QS} = (\vec{a} - \vec{c}) \times (-\vec{b})$.	0.5

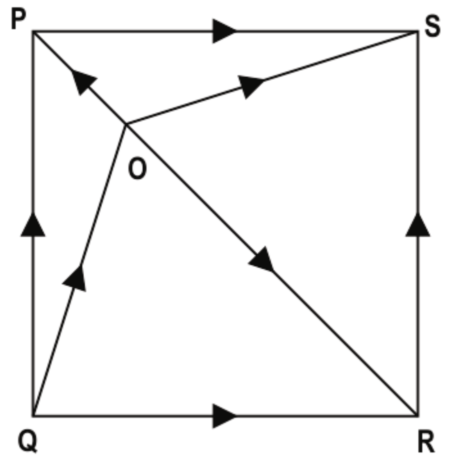
Q.No	Teacher should award marks if students have done the following:	Marks
	Substitutes the above expression in the LHS to prove the given RHS as: $= (\vec{PQ} \times \vec{RS}) + (\vec{QR} \times \vec{PS}) + (\vec{RP} \times \vec{QS})$ $= \{(\vec{b} - \vec{a}) \times (-\vec{c})\} + \{(\vec{c} - \vec{b}) \times (-\vec{a})\} + \{(\vec{a} - \vec{c}) \times (-\vec{b})\}$ $= (\vec{b} \times -\vec{c}) + (-\vec{a} \times -\vec{c}) + (\vec{c} \times -\vec{a}) + (-\vec{b} \times -\vec{a}) + (\vec{a} \times -\vec{b}) + (-\vec{c} \times -\vec{b})$ $= 2(\vec{c} \times \vec{b}) + 2(\vec{a} \times \vec{c}) + 2(\vec{b} \times \vec{a})$ $= 2(\vec{SR} \times \vec{SQ} + \vec{SP} \times \vec{SR} + \vec{SQ} \times \vec{SP})$ $= RHS$	1.5
16	**Finds $\overrightarrow{RS}$ as:** $(-2 + 1)\hat{i} + (5 - 2)\hat{j} + (-1 - 3)\hat{k} = -\hat{i} + 3\hat{j} - 4\hat{k}$	0.5
	Finds $\overrightarrow{RU}$ as: $(0 + 1)\hat{i} + (0 - 2)\hat{j} + (3 - 3)\hat{k} = \hat{i} - 2\hat{j}$	0.5
	Uses the cross product of the above two vectors as follows: $\overrightarrow{RS} \times \overrightarrow{RU} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 3 & -4 \\ 1 & -2 & 0 \end{bmatrix} = -8\hat{i} - 4\hat{j} - \hat{k}$	1.5
	Finds the area of the parallelogram RSTU as: $\left\|\overrightarrow{RS} \times \overrightarrow{RU}\right\| = \sqrt{64 + 16 + 1} = 9$ sq units	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
17	i) Finds v_1, the scalar component along the x -axis as follows: $\cos\frac{\pi}{6} = \frac{v_1}{\lvert\vec{v}\rvert}$ $\Rightarrow v_1 = \frac{\sqrt{3}}{2} \times 2 = \sqrt{3}$	0.5
	Finds v_2, the scalar component along the y -axis as follows: $\cos\theta = \frac{v_2}{\lvert\vec{v}\rvert}$ $\Rightarrow v_2 = 2\cos\theta$	0.5
	Finds v_3, the scalar component along the z -axis as follows: $\cos\frac{\pi}{3} = \frac{v_3}{\lvert\vec{v}\rvert}$ $\Rightarrow v_3 = \frac{1}{2} \times 2 = 1$	0.5
	Uses above three steps and $\lvert\vec{v}\rvert = 2$ to find θ as: $\sqrt{v_1^2 + v_2^2 + v_3^2} = 2$ $\Rightarrow (\sqrt{3})^2 + 4\cos^2\theta + 1 = 4$ $\Rightarrow \cos^2\theta = 0$ $\Rightarrow \theta = 90° \text{ or } \frac{\pi}{2}$	1

Q.No	Teacher should award marks if students have done the following:	Marks
	ii) Writes $\vec{v}$ in its component form as: $= v_1\hat{i} + v_2\hat{j} + v_3\hat{k}$ $= \sqrt{3}\,\hat{i} + 2\cos\frac{\pi}{2}\,\hat{j} + \hat{k}$ $= \sqrt{3}\,\hat{i} + \hat{k}$	0.5
18	**Draws the vector representation of rhombus PQRS by taking RS and QP as vector *a*, SP and RQ as vector *b*. The diagram may look as follows:** 	0.5
	Finds $\overrightarrow{RX}$ and $\overrightarrow{RY}$ as: $\overrightarrow{RX} = \vec{a} + \frac{\vec{b}}{2}$ $\overrightarrow{RY} = \vec{b} + \frac{\vec{a}}{2}$	1

Q.No	Teacher should award marks if students have done the following:	Marks
	Finds $\overrightarrow{RX}$ and $\overrightarrow{RY}$ as: $\vec{a} + \frac{\vec{b}}{2} + \vec{b} + \frac{\vec{a}}{2} = \frac{3}{2}(\vec{a} + \vec{b})$	0.5
	Finds $\overrightarrow{RP}$ as $\overrightarrow{RS} + \overrightarrow{SP} = (\vec{a} + \vec{b})$	0.5
	Uses steps 3 and 4 to conclude that: $\overrightarrow{RX} + \overrightarrow{RY} = \frac{3}{2}\overrightarrow{RP}$	0.5
19	**i) Uses the addition law of vectors to find the position vector of the point Q as:** $\vec{a} + \vec{b}$	0.5
	Finds the position vector of the point U as: $\frac{\vec{a}}{2}$	0.5
	Assumes that V divides PU in the ratio *m* :1 and uses the section formula to write the position vector of the point V in the ratio *m* :1 as: $\overrightarrow{SV} = \frac{m\left(\frac{\vec{a}}{2}\right) + 1(\vec{b})}{m + 1} = \frac{m\vec{a} + 2\vec{b}}{2(m + 1)}$ **(Award 0.5 marks if only the section formula in vector form is correctly written.)**	1

Q.No	Teacher should award marks if students have done the following:	Marks
	Assumes that V divides QS in the ratio *n* :1 and uses the section formula to write the position vector of the point V in the ratio *n* :1 as: $\overrightarrow{SV} = \dfrac{n(\vec{0}) + 1(\vec{a}+\vec{b})}{n+1} = \dfrac{\vec{a}+\vec{b}}{(n+1)}$	1
	Argues that the above two vectors represent the position vector of the same point and equates the coefficients of vectors *a* and *b* to get: 1) $\frac{m}{2(m+1)} = \frac{1}{n+1}$ and 2) $\frac{1}{m+1} = \frac{1}{n+1}$ Writes that $m = n$ and concludes that V divides PU and QS in the same ratio.	1
	ii) Substitutes $m = n$ in equation 1) of the above step to get $m = n = 2$. Concludes that the ratio is 2:1.	1
20	Writes that, $\overrightarrow{AE} + \overrightarrow{FB} + \overrightarrow{CG} + \overrightarrow{HD} = 2\overrightarrow{AO} + 2\overrightarrow{OB} + 2\overrightarrow{CO} + 2\overrightarrow{OD}$.	1
	Simplifies the above equation as: $\overrightarrow{AE} + \overrightarrow{FB} + \overrightarrow{CG} + \overrightarrow{HD} = 2(\overrightarrow{AO} + \overrightarrow{OB}) + 2(\overrightarrow{CO} + \overrightarrow{OD})$ $= 2(\overrightarrow{AB}) + 2(\overrightarrow{CD})$ $= 2(\overrightarrow{AB} + \overrightarrow{CD})$	1

Q.No	Teacher should award marks if students have done the following:	Marks
	Adds and subtracts $\overrightarrow{BC}$ on the RHS of the above equation to get: $\overrightarrow{AE} + \overrightarrow{FB} + \overrightarrow{CG} + \overrightarrow{HD} = 2(\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} - \overrightarrow{BC})$	1
	Simplifies the above equation using the triangular law to get: $\overrightarrow{AE} + \overrightarrow{FB} + \overrightarrow{CG} + \overrightarrow{HD} = 2(\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} - \overrightarrow{BC})$ $= 2(\overrightarrow{AC} + \overrightarrow{CD} - \overrightarrow{BC})$ $= 2(\overrightarrow{AD} - \overrightarrow{BC})$	2
21	**Draws the vector diagram that represents the given scenario. The figure may look as follows:**  	1
	Rearranges the given equation as: $\overrightarrow{OS} - \overrightarrow{OP} = \overrightarrow{QO} + \overrightarrow{OR}$ $\Rightarrow \overrightarrow{OS} + \overrightarrow{PO} = \overrightarrow{QO} + \overrightarrow{OR}$	1

Q.No	Teacher should award marks if students have done the following:	Marks
	Uses vector addition and writes: $\overrightarrow{PS} = \overrightarrow{QR}$ $\Rightarrow$ **PS = QR and PS || QR**	1
	Similarly, concludes that: $\overrightarrow{SR} = \overrightarrow{PQ}$ $\Rightarrow$ **SR = PQ and SR || PQ**	1
	Uses steps 3, 4 and ∠Q = 90° to write ∠Q = ∠P = ∠R = ∠S = 90°.	0.5
	Uses steps 3, 4 and 5 to conclude that PQRS is a square.	0.5
22	Finds $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 3\hat{i} + 3\hat{j} + 4\hat{k}$.	0.5
	Finds $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = 6\hat{i} + 6\hat{j} + 8\hat{k}$.	0.5
	Writes that $\frac{3}{6} = \frac{3}{6} = \frac{4}{8} = \frac{1}{2}$. Hence, $\overrightarrow{AC} = 2\overrightarrow{AB}$. Concludes that the stars with position vectors A, B and C are collinear.	1

Q.No	Teacher should award marks if students have done the following:	Marks
23	Expresses the vector $\overrightarrow{AD}$ in terms of its components as: $\overrightarrow{OD} - \overrightarrow{OA} = 0\hat{i} + 0\hat{j} - 10\hat{k}$	0.5
	Finds the distance between the stars A and D as √100 = 10 units = 10 light years.	0.5
24	**Finds the direction cosines of the star as:** $l = \cos 45° = \frac{1}{\sqrt{2}}$ $m = \cos 60° = \frac{1}{2}$ **where, l and m denotes the direction cosines with respect to the x -axis and y -axis respectively.**	0.5
	Uses the relation $l^2 + m^2 + n^2 = 1$ and finds the direction cosine with respect to the z -axis, n, as: $\frac{1}{2} + \frac{1}{4} + n^2 = 1$ $\Rightarrow n = \frac{1}{2}$ or $\frac{-1}{2}$	0.5
	Uses the given information along with the above steps and finds the position vector of the required star as: $\overrightarrow{OP} = \left\|\overrightarrow{OP}\right\|(l\hat{i} + m\hat{j} + n\hat{k})$ $= 2\left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{2}\hat{j} \pm \frac{1}{2}\hat{k}\right)$ $= (\sqrt{2}\hat{i} + \hat{j} \pm \hat{k})$	1

Chapter - 11
Three Dimensional Geometry

Q: 1 **A line passes through (2, -1, -3) and (0, 2, 3).**

Which of the following are the direction cosines of a line parallel to the given line at a distance of 3 units from the given line?

1. $\frac{2}{7}$, $\frac{1}{7}$ and $\frac{0}{7}$
2. $\frac{-2}{7}$, $\frac{3}{7}$ and $\frac{6}{7}$
3. $\frac{2}{49}$, $\frac{1}{49}$ and $\frac{0}{49}$
4. $\frac{-2}{7}$ + 3, $\frac{3}{7}$ + 3 and $\frac{6}{7}$ + 3

Q: 2 **The direction ratios of two parallel lines are 3, *λ*, (-10) and (-5), 7, *δ* respectively. Which of the following are the values of *λ* and *δ*?**

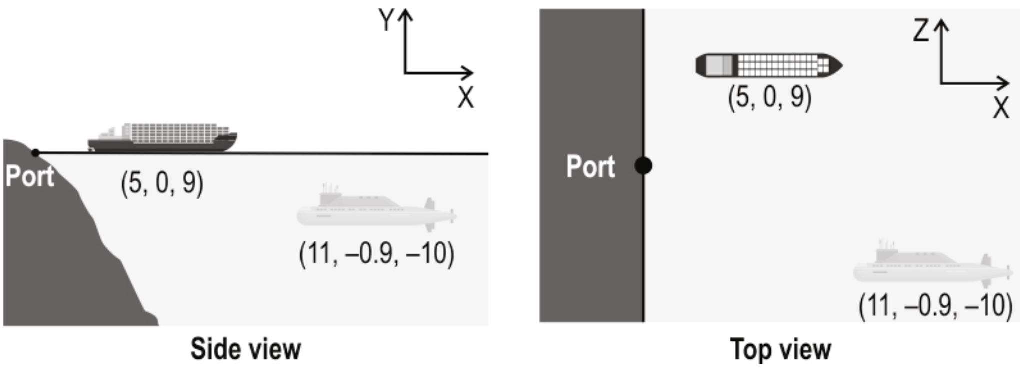
1. $\lambda = \frac{-35}{3}, \delta = 6$
2. $\lambda = \frac{3}{-10}, \delta = \frac{-5}{7}$
3. $\lambda = \frac{-21}{5}, \delta = \frac{50}{3}$
4. (cannot be found without knowing the distance between the two lines.)

Q: 3 **Which of the following is the sum of the direction cosines of the *y*-axis?**

1. 0
2. 1
3. 180°
4. (cannot say without being given two points on the *y*-axis)

Read the information given below and answer the questions that follow.

A supply ship left a port to replenish food and equipment for the Indian navy. Enemies used a submarine to track the ship and scout the port.



(Note: The figure is not to scale.)
Assuming the port to be at the origin, the position of the ship is (5 km, 0 km, 9 km) and the position of the submarine is (11 km, -0.9 km, -10 km).

Q: 4 **What angle does the line joining the ship and the port make with the *z*-axis?**

1. $\cos^{-1} 0$
2. $\cos^{-1} \pm 1$
3. $\cos^{-1} \pm(\frac{9}{\sqrt{106}})$
4. $\cos^{-1} \pm(\frac{10}{\sqrt{302}})$

Q: 5 **Which of the following is the cartesian equation of the line joining the ship to the submarine?**

1. $\frac{x}{6} = \frac{y}{-0.9} = \frac{z}{-19}$
2. $\frac{x-5}{6} = \frac{y}{-0.9} = \frac{z-9}{-19}$
3. $\frac{x-11}{6} = \frac{y+0.9}{-0.9} = \frac{z+10}{-19}$
4. $\frac{x-5}{16} = \frac{y}{-0.9} = \frac{z-9}{-1}$

Q: 6 **The submarine squad wants to find the direction cosines of the line joining the submarine to the port.**

The captain of the submarine says that the direction cosines are:

$\frac{-11}{\sqrt{221.81}}, \frac{0.9}{\sqrt{221.81}}, \frac{10}{\sqrt{221.81}}$

His subordinate says that the direction cosines are:

$\frac{11}{\sqrt{221.81}}, \frac{-0.9}{\sqrt{221.81}}, \frac{-10}{\sqrt{221.81}}$

Who is correct?

1. The captain
2. The subordinate
3. Both of them
4. Neither of them

Q: 7 **The distance between the port and the ship is √106 km, the distance between the submarine and the port is √221.81 km and the distance between the ship and the submarine is √397.81 km.**

Which of the following represents the angle between the lines joining the submarine to the ship and the line joining the submarine to the port?

1. $\cos^{-1}\left|\frac{-35}{\sqrt{106}\times\sqrt{221.81}}\right|$
2. $\cos^{-1}\left|\frac{201}{\sqrt{397.81}\times\sqrt{106}}\right|$
3. $\cos^{-1}\left|\frac{256.81}{\sqrt{397.81}\times\sqrt{221.81}}\right|$
4. $\cos^{-1}\left|\frac{2040}{\sqrt{397.81}\times\sqrt{106}\times\sqrt{221.81}}\right|$

Q: 8 **Which of the following is the equation of the line joining the submarine and the port?**

Equation 1 : $11\hat{i} - 0.9\hat{j} - 10\hat{k} + \lambda(-11\hat{i} + 0.9\hat{j} + 10\hat{k})$
Equation 2 : $\lambda(-11\hat{i} - 0.9\hat{j} - 10\hat{k})$

1. Equation 1
2. Equation 2
3. Both equation 1 and equation 2
4. Neither equation 1 nor equation 2

Q: 9 **Given below are two parallel lines.** **[1]**

$\frac{x+1}{2} = \frac{y-2}{3} = \frac{z+1}{-j}$ and $\frac{x+3}{k} = \frac{y+1}{-2} = \frac{z-3}{-2}$

Find the values of *k* and *j*. Show your work.

Q: 10 **The direction cosines of a line are $\frac{3}{\sqrt{98}}$, $\frac{-5}{\sqrt{98}}$, α.** **[2]**

Find the angle between the line and the *z* -axis. Show your steps and give a valid reason.

Q: 11 **The pair of lines given below are perpendicular to each other.** **[2]**

$$\vec{r} = 2\hat{i} + 3\hat{j} + 7\hat{k} + \lambda(-\hat{i} + \hat{j} + \hat{k})$$
$$\vec{s} = 5\hat{i} - 3\hat{k} + \beta(2\hat{i} - \hat{k} + G\hat{j})$$

Find the value of G. Show your steps.

Q: 12 **The distance between a point P and (-1, 2, 0) is 6√11 units. P also lies on the following line $\frac{x+1}{-3} = y - 2 = z$.** **[3]**

Find the coordinates of P. Show your steps.

Q: 13 **[3]**

Two helicopters flying to Kedar Hills are moving in straight lines represented by $2\hat{i} + 3\hat{j} + 2\hat{k} + (3\hat{i} + \hat{j} + 2\hat{k})$ and $\hat{i} + 2\hat{j} + 3\hat{k} + (3\hat{i} + \hat{j} + 2\hat{k})$ respectively.

Find the shortest possible distance between the helicopters during the flight. Show your steps and give a valid reason.

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	2
2	3
3	2
4	3
5	2
6	3
7	3
8	1

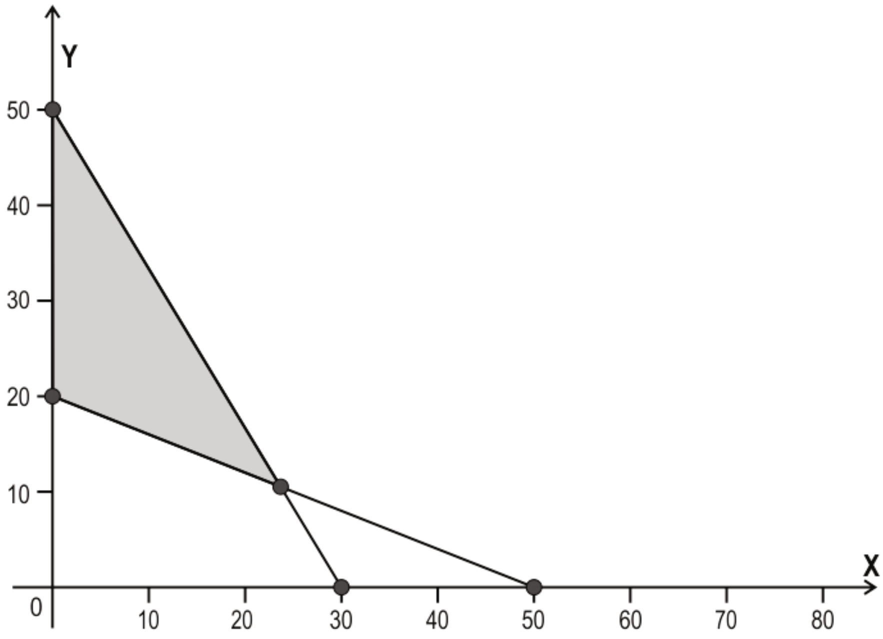
Q.No		Marks
9	**Writes that the lines are parallel and so $\frac{2}{k} = \frac{3}{-2} = \frac{-j}{-2}$.**	**0.5**
	Finds *k* as $\frac{-4}{3}$ and *j* as (-3).	**0.5**
10	**Writes that the sum of the squares of the direction cosines is equal to 1 and finds *α* as $\sqrt{1 - (\frac{9}{98} + \frac{25}{98})} = \frac{-8}{\sqrt{98}}$ or $\frac{8}{\sqrt{98}}$.** **(Award 1 mark if only the value of α is found correctly.)**	**1.5**
	Writes that the angle between the line and the *z* -axis is: **$\cos^{-1}\frac{8}{\sqrt{98}}$ or $\cos^{-1}\frac{-8}{\sqrt{98}}$**	**0.5**
11	**Assumes the angle between the two lines as θ and writes:** $\cos\theta = \left\lvert \frac{\vec{b_1}.\vec{b_2}}{\lvert\vec{b_1}\rvert\lvert\vec{b_2}\rvert} \right\rvert$ where, $\vec{b_1} = (-\hat{i} + \hat{j} + \hat{k})$ and $\vec{b_1} = (2\hat{i} + G\hat{j} - \hat{k})$.	**1**
	Substitutes θ as 90° and finds G as: $cos\ 90^\circ = 0 = \left\lvert \frac{(-1)(2)+(1)(G)+(1)(-1)}{\sqrt{3}\sqrt{5+G^2}} \right\rvert$ $\Rightarrow G = 3$	**1**
12	**Writes that the direction ratios of the given line are (*a* , *b* , *c*) = (-3, 1, 1).** **Writes that any point on the given line is:** **(-1 + λ *a* , 2 + λ *b* , 0 + λ *c*) = (-3λ - 1, λ + 2, λ), where λ is a parameter.**	**1**
	Uses the distance formula between (-1, 2, 0) and (-3λ - 1, λ + 2, λ) to find λ as: **$(-3\lambda)^2 + \lambda^2 + \lambda^2 = (6\sqrt{11})^2$** **=> λ = ±6**	**1**

Q.No	Teacher should award marks if students have done the following:	Marks
	Substitutes λ = 6 and (-6) in (-3λ - 1, λ + 2, λ) to get (-19, 8, 6) and (17, -4, -6) respectively. Concludes that the coordinates of P are (-19, 8, 6) or (17, -4, -6).	1
13	Writes that the helicopters are flying parallel to one another and the shortest distance, ***d***, between them is: $d = \left\| \dfrac{\vec{b} \times (\vec{a}_2 - \vec{a}_1)}{\|\vec{b}\|} \right\|$ Where $\vec{a}_1 = 2\hat{i} + 3\hat{j} + 2\hat{k}$, $\vec{a}_2 = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 3\hat{i} + \hat{j} + 2\hat{k}$	1
	Simplifies the above expression as: $\left\| \dfrac{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ -1 & -1 & 1 \end{vmatrix}}{\sqrt{9+1+4}} \right\|$	1
	Evaluates the above expression as $\frac{\sqrt{19}}{\sqrt{7}}$ units.	1

Chapter - 12
Linear Programming

Q: 1 **Opticare Pvt. Ltd. is conducting an analysis of its operational costs, where labour cost is represented as x and the raw material is represented as y. The cost optimization was framed as a linear programming problem (LPP).**

Observe the graph of the feasible region of the cost optimization LPP shaded below.



Which of the following inequalities is one of the constraints of the LPP?

1. $5x + 3y \geq 150$
2. $5y + 3x \leq 150$
3. $2x + 5y \geq 100$
4. $2y + 5x \leq 100$

Q: 2 **A linear programming problem (LPP) along with its constraints is given below.**

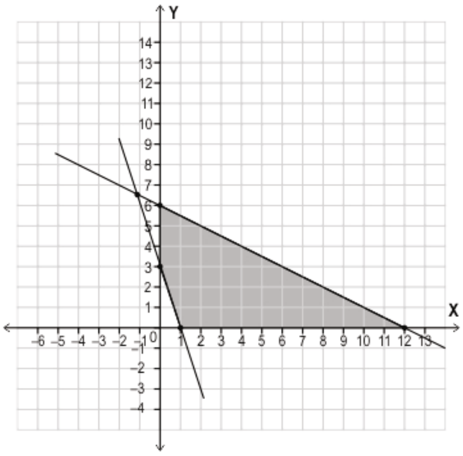
Maximise Z = 2 x + y

subject to the constraints:

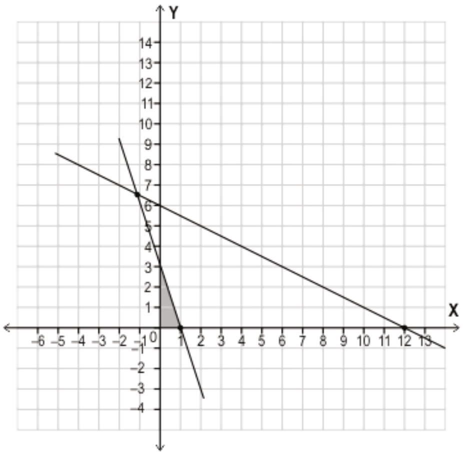
$x + 2y \leq 12$
$3x + y \geq 3$
$x \geq 0$
$y \geq 0$

Which option represents the feasible region of the above LPP?

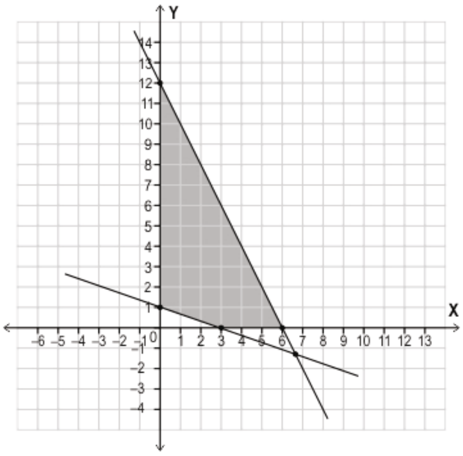
1



2



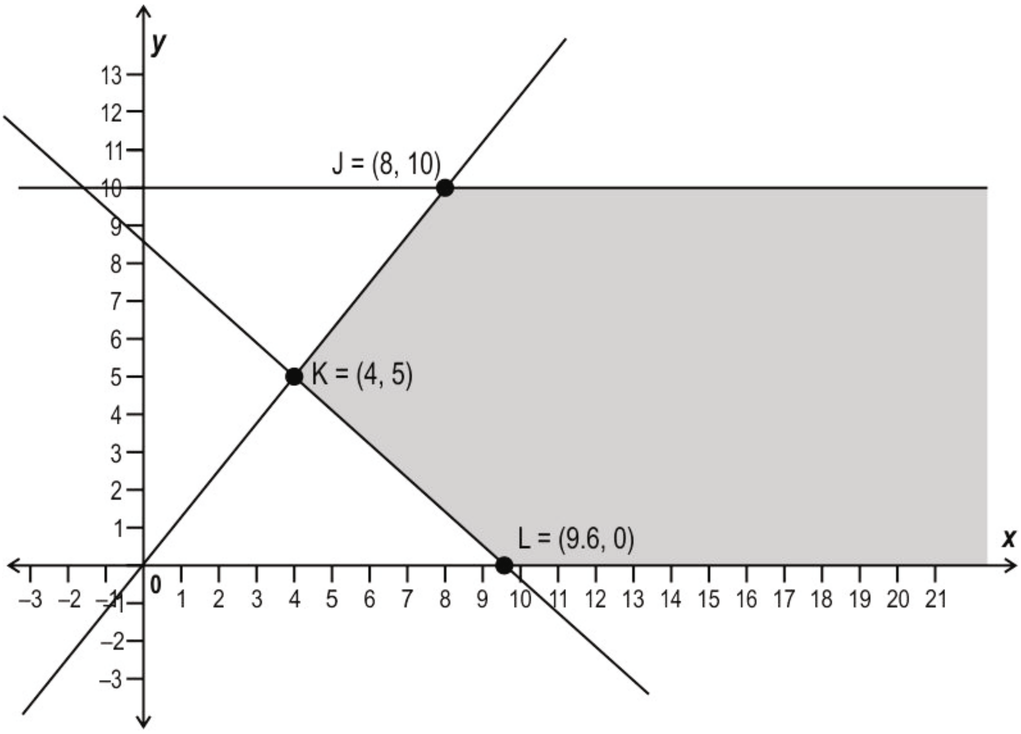
3



4 (there exists no feasible region for the given LPP)

Q: 3 **The objective function of a linear programming problem, along with its feasible region given below.**

Z = 10 *x* + 3 *y*

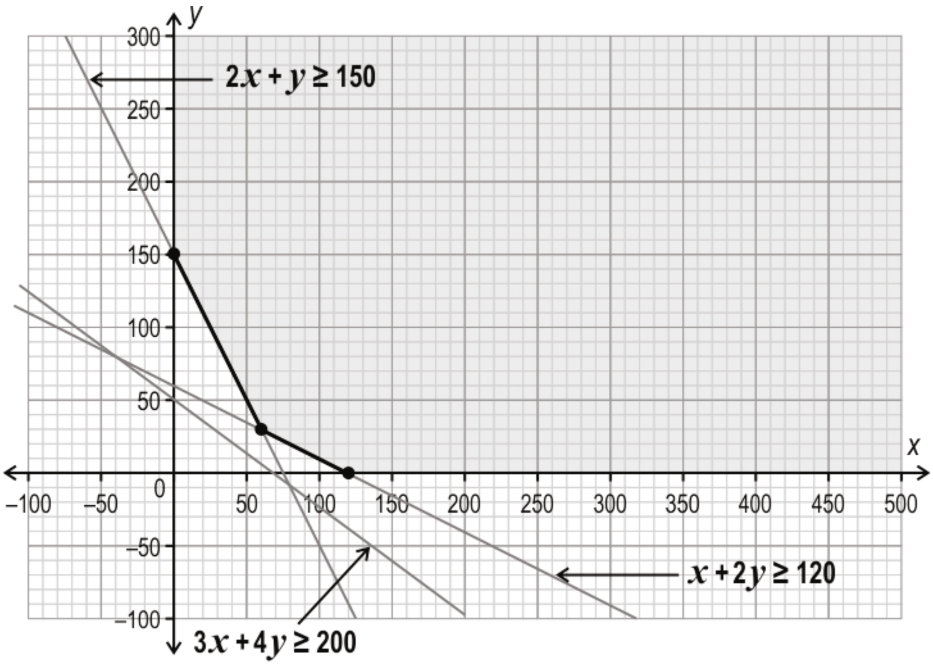


If J, K and L are the corner points, at which point does Z attain its minimum value?

1 Point J
2 Point K
3 Point L
4 (there exists no point at which Z attains its minimum value)

Q: 4 **The objective function of a linear programming problem (LPP), Z = 4x + 3y, has to be minimised.**

The feasible region of this LPP, along with its constraints, is shown in the graph below.



Which constraint, if removed, will not affect the feasible region?

1. $x + 2y \geq 120$
2. $2x + y \geq 150$
3. $3x + 4y \geq 200$
4. (Any of the given constraints, if removed, will affect the feasible region.)

Q: 5 **State whether the following statement is true or false. Justify your answer.** **[1]**

A linear programming problem can have infinitely many optimal solutions.

Q: 6 **A packaging company has the capacity to produce rectangular boxes and circular boxes. Each rectangular box takes 2 minutes to make and it sells for a profit of Rs 4. Each circular box takes 3 minutes to make and it sells for a profit of Rs 5.** **[1]**

Their client requests for 25 boxes to be ready in one hour. The packaging company wants to maximise their profit from this order.

Frame this optimisation problem as a linear programming problem.

Q: 7 **State whether the following statement is true or false. Justify your answer.** **[1]**

A linear programming problem whose feasible region is unbounded does not have an optimal solution.

Q: 8 The objective function of a linear programming problem is given by Z = a x + b y ; where a and b are constants. If the minimum of Z occurs at two points, (50, 30) and (20, 40), find the relationship between a and b. Show your work. **[2]**

Q: 9 Sameer framed the following linear programming problem to minimise the monthly operational cost in running his bakery. **[2]**

Minimise Z = 200 x + 300 y

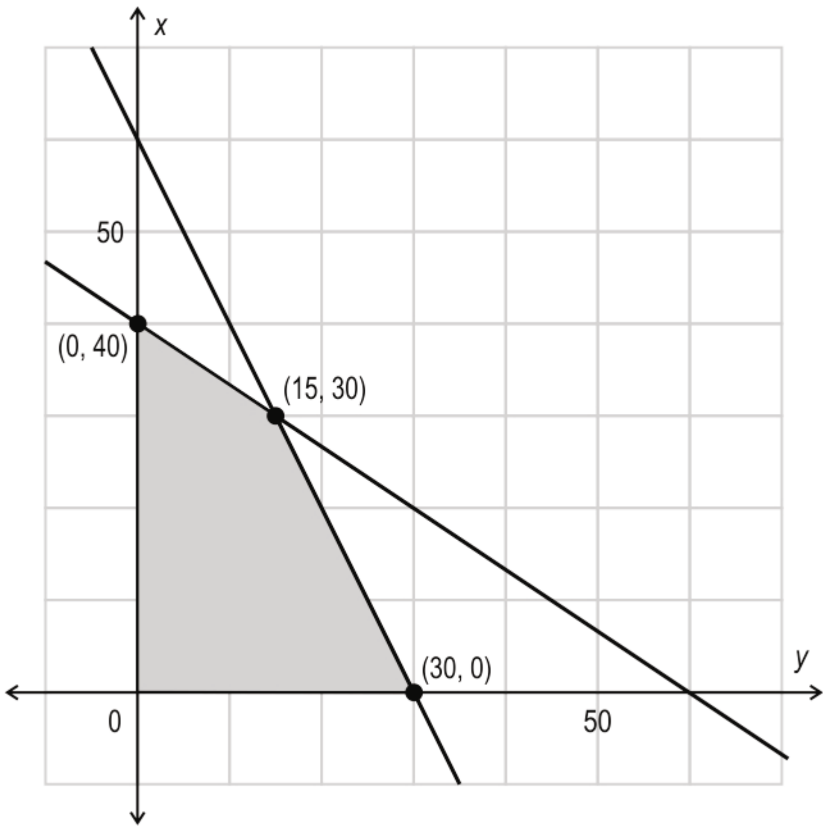
subject to the constraints:

$2x + 3y \geq 1200$
$1.5y + 2x \geq 900$
$x + y \leq 400$
$x, y \geq 0$

where x is the number of orders for bread loaves and y is the number of orders for cakes. Graph the feasible region for Sameer's LPP and find the optimal solution.

Q: 10 **The shaded region in the graph shown below represents the feasible region for a linear programming problem. The objective function of the LPP is:** **[2]**

Maximize Z = 5 x + 3 y

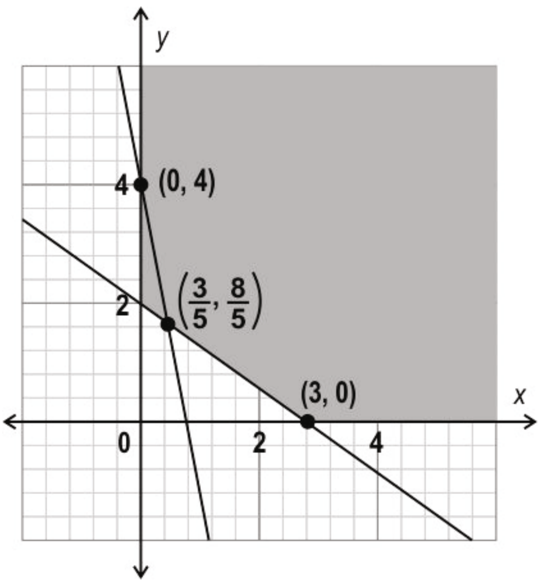


Find all the constraints for the above LPP. Show your work.

Q: 11 **A linear programming problem (LPP) along with the graph of its constraints is shown below. The shaded portion represents the feasible region.** **[2]**

Maximise: Z = 10 x + 20 y

Subject to:
$2x + 3y \geq 6$
$4x + y \geq 4$
$x \geq 0, y \geq 0$



What can you conclude about the existence of optimal solution for the above LPP? Justify your answer.

Q: 12 **A consultancy firm is preparing an office trip for its 400 employees. The bus rental company has 2 options for the buses that they can offer along with a maximum of 9 drivers. The capacity and rental costs are shown below.** **[3]**

Bus type	Maximum capacity	Rental cost (in Rs.)
Large	50	3000
Medium	40	2000

Determine the number of buses that the consultancy firm should book to minimise the total rental cost. Sketch the feasible region and show your steps.

Q: 13 **Ritvik owns two wholesale stores located in Noida and Gurgaon that supply shoes to retail stores in Dwarka, Saket and Greater Kailash in Delhi. The retail stores require 250, 350 and 400 pairs of shoes respectively in a month.** **[3]**

The distribution capacities of the wholesale stores at Noida and Gurgaon are 450 and 500 pairs respectively per month. The cost of transportation per pair is given below:

From	To Dwarka (in Rs)	To Saket (in Rs)	To Greater Kailash (in Rs)
Noida	10	15	12
Gurgaon	12	15	10

Assuming that the wholesale stores operate at their maximum distribution capacities, frame an objective function to minimise the cost of transportation of the shoes from the two wholesale stores to the three retail stores.

Q: 14 **Shaanta is a wholesale dry fruit trader who deals with figs and cashews.** **[5]**

The maximum capital available with her in a certain month is Rs 24,00,000. She has a warehouse with a maximum capacity to store 50 quintals of dry fruits at a time. Figs cost her Rs 40,000 per quintal and cashews Rs 60,000 per quintal. She earns a profit of Rs 2,000 per quintal on figs and Rs 3,000 per quintal on cashews.

How many quintals of figs and cashews should she purchase that month to make a maximum profit? Show your steps.

Q: 15 **A seller wants to pack Ajwa dates and Omani dates such that, in each packet, the combined weight is at most 15 kg and the weight of Omani dates is at most twice the weight of Ajwa dates. The seller anticipates a profit of Rs 55 per kg on Ajwa dates and Rs 70 per kg on Omani dates.** **[5]**

How many kilograms of Ajwa dates and Omani dates should be packed in each packet to attain maximum profit? Show your steps.

Answer the questions based on the given information.

Star Auto Private Limited (SAPL), an automobile company, manufactures and supplies precision machine components. Among other products, two products manufactured by SAPL are - Roller Bush and Hydraulic Valve shown below. These products are manufactured and packed in boxes of 25.

Each box of Roller Bush requires 2 kg of aluminium and 1 kg of steel. Each box of Hydraulic Valve requires 14 kg of aluminium and 2 kg of steel. SAPL has an available supply of 70 kg of aluminium and 20 kg of steel per day to manufacture these two products. The plant makes a profit of Rs 20 on each box of Roller Bush and Rs 50 on each box of Hydraulic Valve.

Q: 16 **Write the mathematical formulation of the given scenario that gives the number of units of Roller Bush and Hydraulic Valve that SAPL must manufacture in order to maximize its profit.** **[1]**

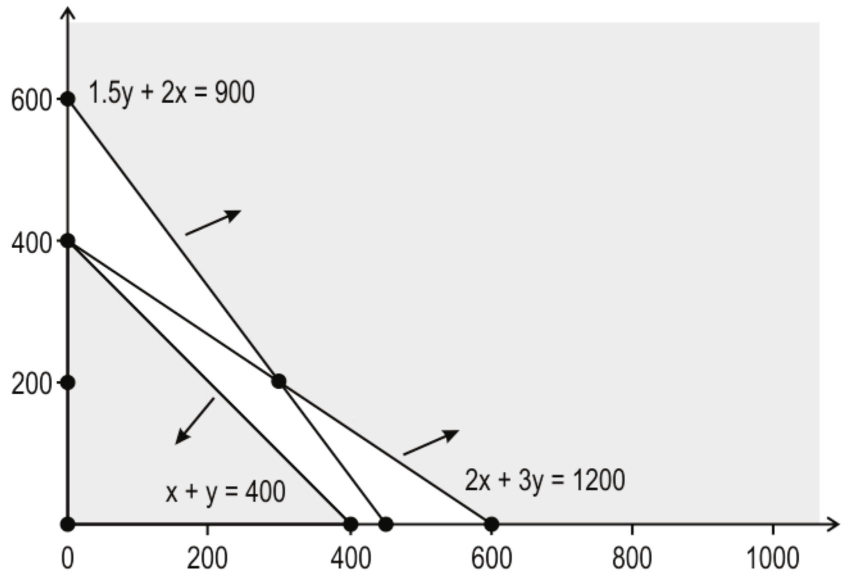
Q: 17 **Determine the feasible region of the linear programming problem defined by the above scenario graphically.** **[2]**

Q: 18 **Determine the number of units of Roller Bush and Hydraulic Valve that SAPL must manufacture in order to maximise its profit. Show your work.** **[2]**

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	3
2	1
3	2
4	3

Q.No	Teacher should award marks if students have done the following:	Marks
5	Writes true.	0.5
	Justifies that in certain linear programming problems, the maximum/minimum value is obtained at 2 distinct corner points of the feasible region. In such cases, every point on the line joining those two points will be an optimal solution. Hence, a linear programming problem can have infinitely many optimal solutions. (Award full marks for any equivalent justification.)	0.5
6	Writes the entire LPP as: Maximise Z = 4 r + 5 c subject to the constraints: $r + c = 25$ $2r + 3c \leq 60$ $r \geq 0$ $c \geq 0$ where, r = number of rectangular boxes c = number of circular boxes (Award 0.5 mark if either the objective function or constraints are correct, but not both.)	1
7	Writes False(F).	0.5
	Writes that, if on graphing the objective function, the open half-plane does not have any points common with the feasible region then the linear programming problem has an optimal solution.	0.5
8	Writes that, since the minimum of Z occurs at both (50, 30) and (20, 40), 50a + 30b = 20a + 40b.	1
	Solves the above equation to find the relationship between a and b as 3a = b.	1

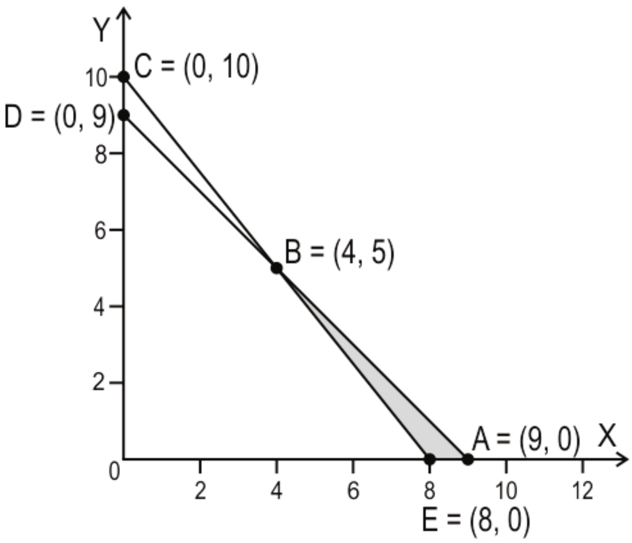
Q.No	Teacher should award marks if students have done the following:	Marks
9	Graphs the constraints of the LPP as: 	1
	Argues that, since no overlapping region exists for the LPP, there is no optimal solution in this case.	1
10	Uses the given points on the graph to find the following constraints using any suitable method: $2x + 3y \leq 120$ $2x + y \leq 60$	1.5
	Writes the remaining constraints by observing the graph of the feasible region as: $x \geq 0$ $y \geq 0$	0.5
11	Writes that the given LPP has no maximum value/no optimal solution.	0.5
	Justifies the answer by evaluating the objective function $Z = 10x + 20y$ at each corner point and finding the largest value of Z as 80, which is obtained at the point (0, 4).	1

<table>
<tr><th>Q.No</th><th></th><th>Marks</th></tr>
<tr><td></td><td>Writes that the open plane determined by 10 x + 20 y > 80 has points in common with the feasible region and hence the given LPP has no maximum value.</td><td>0.5</td></tr>
<tr><td>12</td><td>Assumes the number of large buses as x and medium buses as y. Writes the LPP as:

Minimise Z = 3000 x + 2000 y

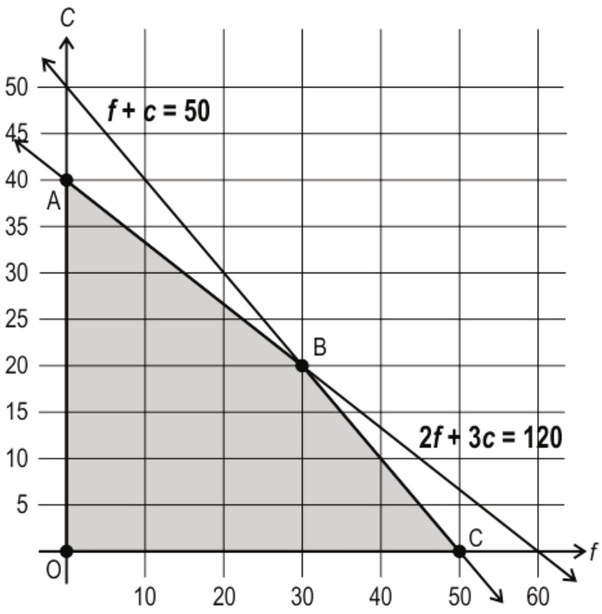
subject to the constraints:

50 x + 40 y ≥ 400
x + y ≤ 9
x ≥ 0
y ≥ 0</td><td>0.5</td></tr>
<tr><td></td><td>Sketches the feasible region as:

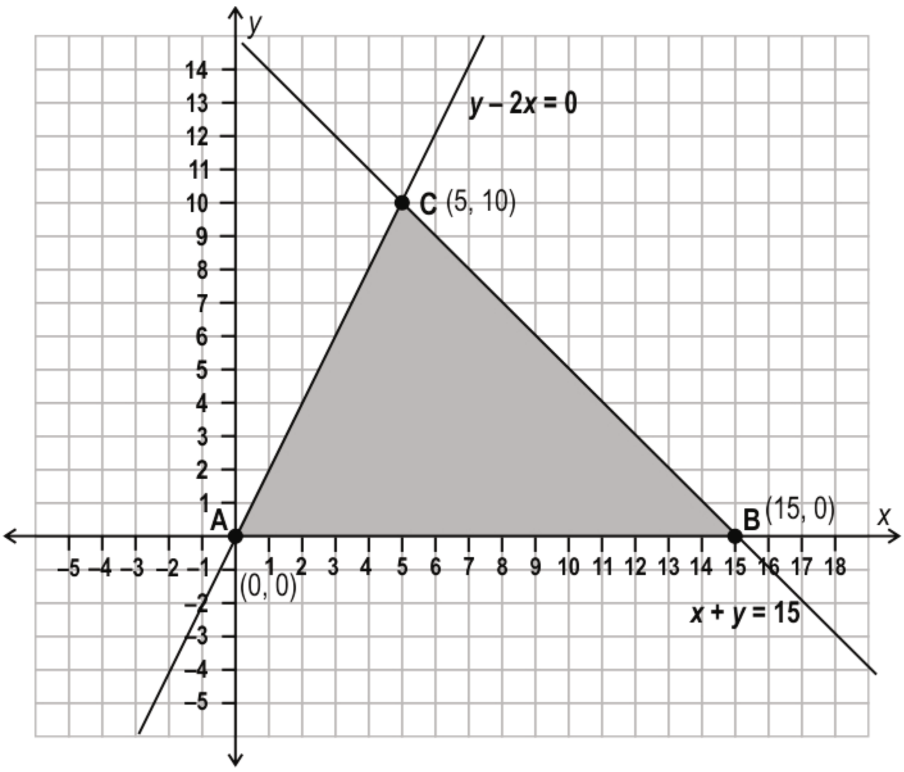

</td><td>1</td></tr>
<tr><td></td><td>Finds the objective function value at the 3 corners of the feasible region as:

<table><tr><th>Vertex</th><th>Objective function value</th></tr><tr><td>(4, 5)</td><td>Rs 22000</td></tr><tr><td>(8, 0)</td><td>Rs 24000</td></tr><tr><td>(9, 0)</td><td>Rs 27000</td></tr></table></td><td>1</td></tr>
</table>

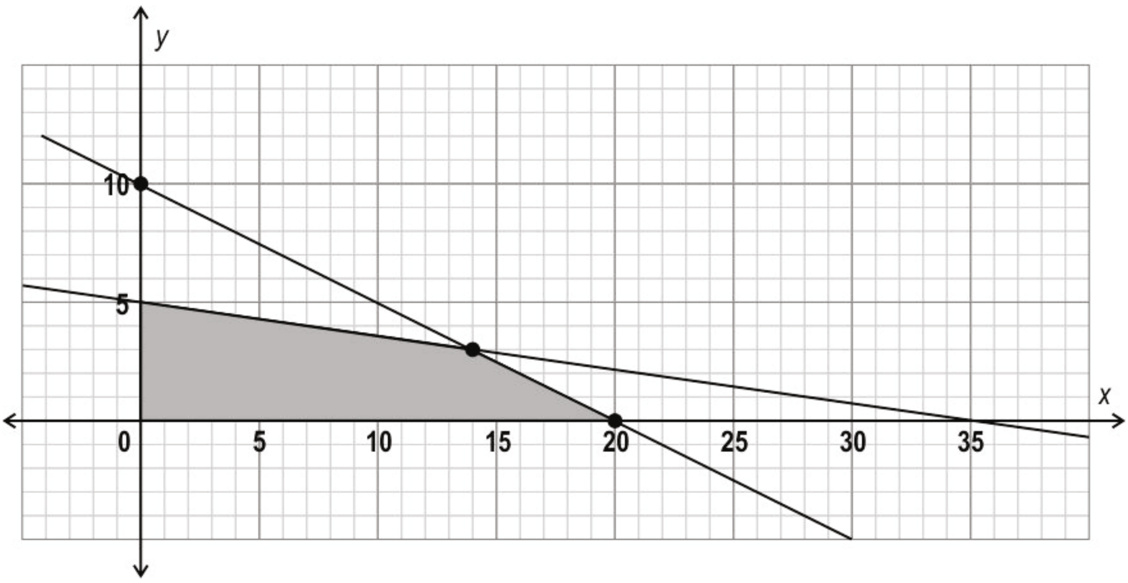
Q.No	Teacher should award marks if students have done the following:	Marks
	Concludes that the consultancy firm should book 4 large buses and 5 medium buses.	0.5
13	Assumes the number of pairs of shoes transported from Noida to Dwarka as x, from Noida to Saket as y and from Noida to Greater Kailash as $(450 - x - y)$.	1
	Finds the number of pairs of shoes transported from Gurgaon to Dwarka as $(250 - x)$, from Gurgaon to Saket as $(350 - y)$ and from Gurgaon to Greater Kailash as $500 - (250 - x + 350 - y)$ or $(x + y - 100)$.	1
	Writes the objective function to minimise the cost of transportation as: Minimise $Z = 10x + 15y + 12(450 - x - y) + 12(250 - x) + 15(350 - y) + 10(x + y - 100)$ or Minimise $Z = 2(6325 - 2x - y)$ (Award full marks if the objective function is framed by taking x and y as number of pairs of shoes transported from Gurgaon to Dwarka and Saket respectively.)	1
14	Assumes the quantity of figs and cashews to be f and c quintals respectively. Frames the objective function of the given problem as: Maximise $Z = 2{,}000f + 3{,}000c$	0.5
	Writes the constraints of the given problem as: $f + c \leq 50$ $40{,}000f + 60{,}000c \leq 24{,}00{,}000$ or $2f + 3c \leq 120$ $f \geq 0, c \geq 0$	1

Q.No	Teacher should award marks if students have done the following:	Marks
	Draws the graph of the system of inequalities and shades the feasible region. The graph may look as follows:  **(Award 0.5 marks each for drawing the correct line and 0.5 mark for shading the feasible region correctly.)**	**1.5**
	Evaluates the objective function at the corner points to find the optimal solution as follows:	**1**
	Writes that the maximum value of Z is obtained at both the corner points A and B. Hence, concludes that every point on the line segment joining A and B is an optimal solution and Shaanta can buy any combination of weights corresponding to a point on the line segment joining A and B.	**1**

Corner points	**Z = 2000 *f* + 3000 *c***
O (0, 0)	0
A (0, 40)	1,20,000
B (30, 20)	1,20,000
C (50, 0)	1,00,000

Q.No	Teacher should award marks if students have done the following:	Marks
15	**Writes the mathematical formulation of the given problem as follows:** **Maximise Z = 55 x + 70 y** **subject to the constraints:** $y - 2x \leq 0$ $x + y \leq 15$ $x \geq 0$ $y \geq 0$ **where x and y are the weights of Ajwa dates and Omani dates (in kgs) in each packet respectively.**	2
	Draws the graph of the system of inequalities and shades the feasible region. The graph may look as follows:  	1.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Evaluates the objective function at the corner points to find the optimal solution as follows: <table><tr><th>Corner points</th><th>Z = 55 x + 70 y</th></tr><tr><td>A (0, 0)</td><td>0</td></tr><tr><td>B (15, 0)</td><td>825</td></tr><tr><td>C (5, 10)</td><td>975</td></tr></table>	1
	Uses the above step and writes that the maximum profit that can be attained on one packet is Rs 975 when 5 kgs of Ajwa dates and 10 kg of Omani dates are packed in each packet.	0.5
16	**Assumes the number of units of Roller Bush and Hydraulic Valve to be manufactured as x and y respectively and writes the mathematical formulation of the above scenario as:** **Maximise Z = 20 x + 50 y** **Subject to constraints:** $2x + 14y \leq 70$ $x + 2y \leq 20$ $x \geq 0$ $y \geq 0$	1

<table>
<tr><th>Q.No</th><th>Teacher should award marks if students have done the following:</th><th>Marks</th></tr>
<tr><td>17</td><td>Plots the graph of the constraints and shades the feasible region as follows:


(Award only 1 mark if the constraints are plotted correctly but feasible region is not shaded/shaded incorrectly.)</td><td>2</td></tr>
<tr><td>18</td><td>Identifies the corner points and evaluates the value of Z at those points as follows:
<table><tr><th>Corner point</th><th>Z = 20 x + 50 y</th></tr><tr><td>(0, 0)</td><td>0</td></tr><tr><td>(0, 5)</td><td>250</td></tr><tr><td>(20, 0)</td><td>400</td></tr><tr><td>(14, 3)</td><td>430</td></tr></table>
Writes that 430 is the largest value of Z and is obtained at (14, 3).</td><td>1.5</td></tr>
<tr><td></td><td>Concludes that 14 units of Roller Bush and 3 units of Hydraulic Valve must be manufactured by SAPL in order to maximize its profit.</td><td>0.5</td></tr>
</table>

Chapter - 13
Probability

Q: 1 **Which of the following represents a Bernoulli trial?**

1. Tossing a coin until a tail is obtained.
2. Recording eye colours found in a group of 500 people.
3. Asking a random sample of 50 people if they have ever been to Germany.
4. A bag contains 8 red marbles and 5 blue marbles. A marble is picked from the bag 5 times without replacement.

Q: 2 **Study the following experiments.**

Experiment 1:
A fair coin is flipped 7 times with the objective of getting heads each time. The outcome is heads 5 times out of 7.

Experiment 2:
A pen is selected at random from a box 4 times with the objective of getting a red pen. The box contains 2 blue, 2 red and 2 black pens and the selected pen is replaced after each trial.

In which experiment(s) is/are the trials Bernoulli trials?

1. Experiment 1
2. Experiment 2
3. both Experiment 1 and Experiment 2
4. Neither Experiment 1 nor Experiment 2

Q: 3 **During a job interview, the probability of a candidate having a B.Tech degree was $\frac{5}{12}$ while the probability of a candidate having an MBA degree was $\frac{7}{16}$. The probability of a candidate having a B.Tech degree or an MBA degree or both is $\frac{11}{24}$.**

What is the probability of a candidate having a B.Tech degree given that the candidate has an MBA degree?

1. $\frac{5}{12}$
2. $\frac{19}{21}$
3. $\frac{20}{21}$
4. $\frac{19}{20}$

Q: 4 **A dog has six puppies. Assume that each puppy is equally likely to be a male or a female puppy.**

What is the probability that the dog has exactly 4 female puppies?

1. $\frac{1}{16}$
2. $\frac{15}{64}$
3. $\frac{15}{16}$
4. $\frac{2}{3}$

Q: 5 **M and N are two independent events.**

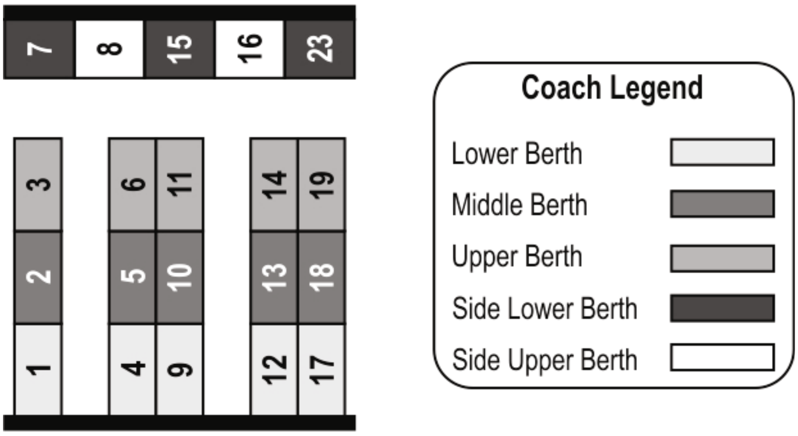
Which of the following statement(s) is/are DEFINITELY true?

i) M and N are mutually exclusive.
ii) The sum of the probabilities of M and N is 1.

1. only (i)
2. only (ii)
3. both (i) and (ii)
4. neither (i) nor (ii)

Use the information given below to solve the problems that follow.

A sleeper train has 5 types of seats. They are lower berth, middle berth, upper berth and side lower berth and side upper berth.



(Note: This picture is only to give you an idea of what a railway coach looks like. It DOES NOT represent the actual number of seats.)

Raj was wait-listed for a train from Dehradun to Delhi on a railway ticket booking application. In case his ticket got confirmed in this train, the probability that he got:

i) side berth (side lower and side upper berth) was, $P(S) = \frac{1}{3}$.
ii) upper berth was, $P(U) = \frac{2}{9}$.
iii) lower berth given that it is the side berth was, $P(L|S) = \frac{1}{4}$.
iv) upper berth or the side berth (side lower and side upper berth) was, $P(U \cup S) = \frac{17}{36}$.
v) middle berth was not 0.

(Note: P(U) does not include the side upper berths.)

Q: 6 **Which of the following statements is true?**

1. The probability of getting a lower berth is $\frac{7}{9}$.
2. The probability of getting a middle berth or side berth is $\frac{7}{9}$.
3. The probability of getting a middle berth or lower berth or side berth is $\frac{7}{9}$.
4. (none of the given statements are correct.)

Q: 7 **Which of the following is the probability of the seat being a side berth as well as a lower berth?**

1. $\frac{1}{12}$
2. $\frac{1}{3}$
3. $\frac{1}{4}$
4. $\frac{7}{12}$

Q: 8 **Which of the following is the probability of Raj getting an upper berth as well as a side berth?**

1. $\frac{1}{12}$
2. $\frac{2}{9}$
3. $\frac{17}{36}$
4. $\frac{5}{9}$

Q: 9 **Which of the following is the probability of Raj getting a side berth given that he has got an upper berth?**

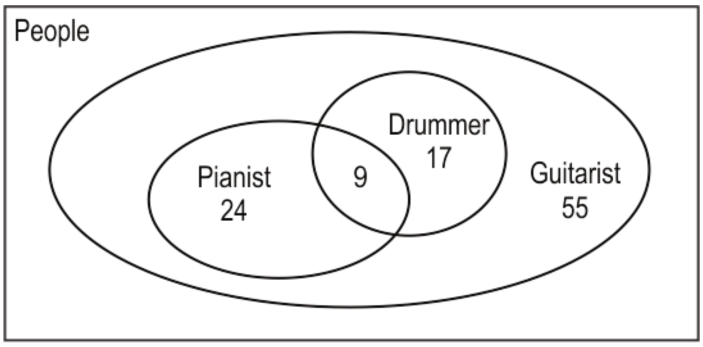
1 $\frac{1}{3}$ **2** $\frac{3}{8}$ **3** $\frac{1}{4}$ **4** $\frac{5}{9}$

Q: 10 **If the probability of Raj buying food on the train given that he got an upper berth is, P(F|U) = $\frac{1}{15}$, what can be said about the probability of Raj buying food given that he got a middle berth, P(F|M)?**

(Note: Getting food on the train and getting a lower, middle or upper berth are two independent events.)

1 P(F|M) > P(F|U) **2** P(F|M) < P(F|U)
3 P(F|M) = P(F|U) **4** (cannot be said without knowing P(M))

Q: 11 **Shown below is a Venn diagram representing the number of people in a music class who play different instruments.** **[1]**



Find the probability that a guitarist selected at random is also a pianist.

Q: 12 **There are three types of seats in an airplane: aisle, middle and window. While booking a ticket, one can pay extra money to reserve a specific seat type.** **[1]**
While booking a ticket on a website for a specific flight one particular day, the following observations were recorded:

i) the probability of a person not paying for a specific seat type was $\frac{1}{6}$.
ii) the probability of getting a window seat given that the person did not pay for a specific seat type was $\frac{1}{12}$.

Find the probability of a person getting a window seat without paying for a specific seat type. Show your work and give a valid reason.

Q: 13 **X and Y are the two events such that P(X|Y) = 0.2 and P(Y) = 0.5.** **[2]**

Find the value of P(X' ∩ Y). Show your steps.

Q: 14 **On a particular day, Vidit is going to play a game of carrom against one of three opponents. All opponents are equally likely to be paired with him. The table below shows the chances of Vidit winning when paired against each opponent.** **[2]**

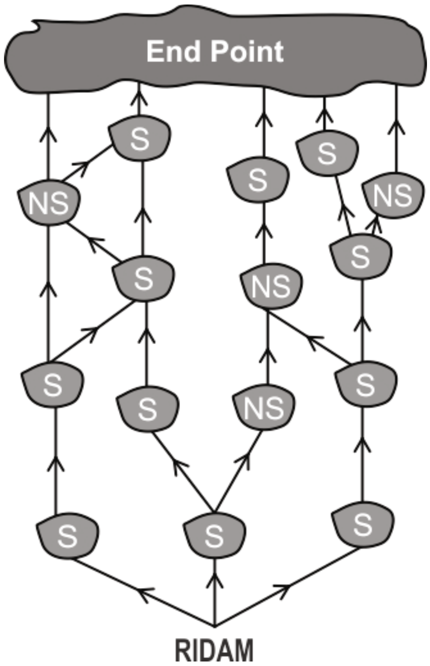
Opponent	Opponent 1	Opponent 2	Opponent 3
Vidit's chances of winning	80%	40%	x

If the probability that Vidit wins the game that day is $\frac{7}{15}$, find the chances of Vidit winning the game when paired with Opponent 3. Show your steps.

Q: 15 **Sukriti is playing a game with 16 identical white balls such that she can paint them as many times as she wants. She has the same set of balls in both the rounds. In the first round, she randomly spray paints 4 balls green. In the second round, she randomly spray paints 8 balls green.** **[2]**

At the end of the second round, what is the probability that a ball chosen at random is white? Show your steps.

Q: 16 Ridam went to a game show where she was asked to cross a puddle using stones. She cannot skip a stone. Shown below is a representation of the game. **[2]**



Stones marked S are stationary while those marked NS will sink as soon as she steps on them. She can only reach the other side of the puddle if she steps on all stationary stones.

If she does not know which stones are stationary, draw a tree diagram and find the probability that she reaches the other side of the puddle. Show your work.

Q: 17 A survey was conducted for all grade 10 students to test their inclination towards opting for science or commerce in grade 11. 75% of class 10 students are fluent in mathematics while the rest are fluent in economics. Based on past data, it is known that: **[2]**

♦ 80% of the students are fluent in business studies if they are fluent in economics.
♦ The probability of a student being fluent in business studies if they are fluent in mathematics is 15%.

If one of the students surveyed was found to be fluent in business studies, use Bayes' theorem to find the probability that the applicant is also fluent in economics? Show your work.

Q: 18 **In a particular week, the probability that it will rain on Monday, Tuesday and Wednesday are $\frac{1}{6}$, $\frac{2}{5}$ and $\frac{4}{5}$ respectively.** **[3]**

What is the probability that it will rain on at least one of the three days? Show your steps.

(Note: Whether it rains or does not rain on one of the days does not affect the probabilities of it raining on the following days.)

Q: 19 **In a particular week, a flight from Jaipur to Shillong is found to have a late arrival on:** **[3]**

♦ $\frac{1}{5}$ **of the occasions if it is on time the previous day and**

♦ $\frac{3}{10}$ **of the occasions if it is late the previous day.**

If it was on time on Wednesday, draw a tree diagram to find the probability that it will be on time on Friday. Show your work.

Q: 20 **A class 6 student, Himansh, has a 60% chance of having a misconception in decimals. His math teacher asks him take a remedial test to check if he a misconception. The test outcome is red if Himansh has the misconception and green if he does not have the misconception. Based on past data for class 6 students, the following points are known:** **[3]**

♦ **If the student has a misconception, the test is 80% accurate.**
♦ **If the student doesn't have a misconception, the test is 90% accurate.**

Find the probability that Himansh has a misconception given his test outcome was green. Show your steps.

Q: 21 **At a magic show, a magician has a sealed box that has a ball. The ball is equally likely to be either black or white. He opens it and without looking, puts a black ball in the box. He then randomly picks one of the two balls from the box.** **[5]**

If he picked a black ball, use Bayes' Theorem to find the probability that the original ball in the box is also black? Show your steps.

Q: 22 **Danish had one hundred cards. Each card had a natural number from 1 to 12 on its face.** **[5]**

Number on the card	1	2	3	4	5	6	7	8	9	10	11	12
Number of cards	5	10	12	8	6	7	6	10	12	15	4	5

He said, "If a card is drawn at random, the probability that the number on the card is even provided that it is a multiple of 3 is the same as the probability that the number on the card is a multiple of 3 provided that it is an even number."

Is Danish right in his conclusion? Justify your answer.

Q: 23 **Sundar has a basket of fruits that contains only 6 guavas and 4 apples. He picks three fruits, one after the other, at random without replacing any of the fruits.** **[5]**

Find the probability that:

i) all three fruits picked are apples.
ii) the first fruit picked is guava and the next two are apples.
iii) at least one of the fruits picked is a guava.

Show your steps.

Read the information and answer the questions that follow.

In a clinic, out of 40 patients who are waiting to see the doctor:

- ♦ 24 have cold
- ♦ 18 have fever
- ♦ 9 have migraine
- ♦ 4 have both cold and fever
- ♦ 6 have both fever and migraine
- ♦ 3 have both migraine and cold
- ♦ 2 have all the three

(Note: Assume that each of the 40 patients present is suffering from one or more of the 3 ailments - cold, fever or migraine only.)

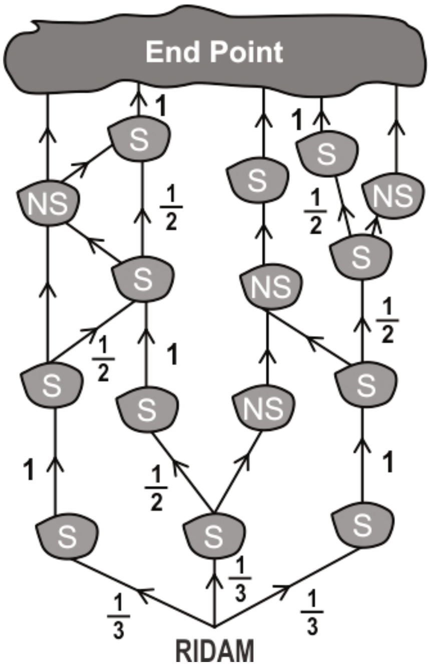
Q: 24 **If a patient is selected at random, find the probability that he/she DOES NOT have migraine given that he/she has fever. Show your work.** **[2]**

Q: 25 **If a patient is selected at random, what is the probability that he/she has cold or fever or both, given that he/she has migraine? Show your work.** **[3]**

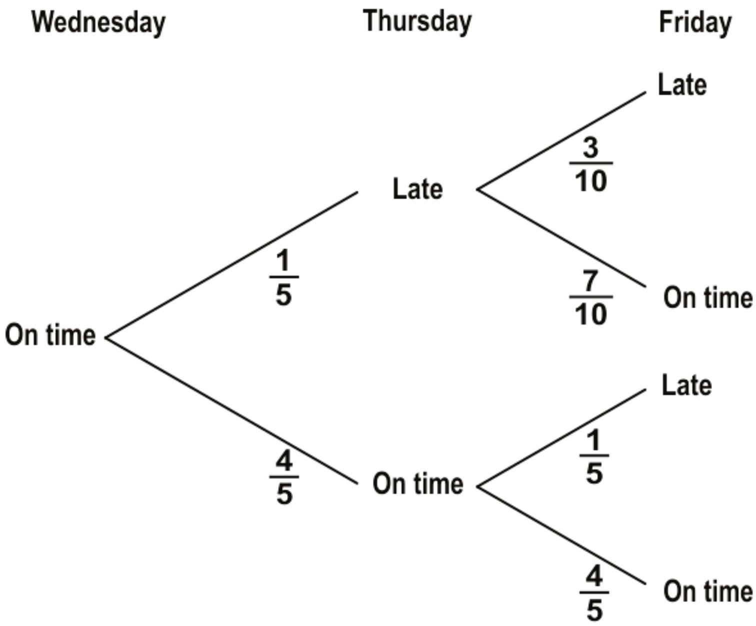
The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	3
2	3
3	2
4	2
5	4
6	3
7	1
8	1
9	2
10	3

Q.No	Teacher should award marks if students have done the following:	Marks
11	**Finds the probability that a guitarist selected at random is also a pianist as:** $P(pianist \mid guitarist)$ $= \frac{P(pianist \cap guitarist)}{P(guitarist)}$ $= \frac{33}{55} = \frac{3}{5}$ **(Award 0.5 marks if only the correct formula for conditional probability is written.)**	**1**
12	**Writes that, by using the multiplication theorem of probability, the probability of getting a window seat without paying for a specific seat type can be found as** $\frac{1}{6} \times \frac{1}{12} = \frac{1}{72}$. **(Award full marks for any other valid reason.)**	**1**
13	**Writes that** $P(X' \cap Y) = P(X'\|Y) \times P(Y)$.	**0.5**
	Uses the property of conditional probability and simplifies the above equation as: $P(X' \cap Y) = [1 - P(X\|Y)] \times P(Y)$	**1**
	Substitutes the given values in the above equation to find $P(X' \cap Y)$ **as:** $P(X' \cap Y) = 0.8 \times 0.5 = 0.4$	**0.5**
14	**Uses the given information and writes the equation for Vidit winning the game that day as:** $\frac{7}{15} = \left(\frac{1}{3} \times \frac{80}{100}\right) + \left(\frac{1}{3} \times \frac{40}{100}\right) + \left(\frac{1}{3} \times \frac{x}{100}\right)$	**1**
	Simplifies the above equation as: $\frac{x}{100} = \frac{7}{5} - \frac{120}{100}$	**0.5**
	Solves the above equation for *x* and finds the chances of Vidit winning the game when paired with Opponent 3 as 20%.	**0.5**

Q.No	Teacher should award marks if students have done the following:	Marks
15	Finds the probability of a ball being white in the first round as $1 - \frac{4}{16} = \frac{3}{4}$.	0.5
	Finds the probability of a ball being white in the second round as $1 - \frac{8}{16} = \frac{1}{2}$.	0.5
	Finds the probability of a ball being white at the end of the second round as $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$.	1
16	Identifies that there are three paths which Ridam can take to successfully reach the other side of the puddle and draws a tree diagram. The diagram may look as follows: 	1
	Finds the probability of Ridam successfully reaching the other side of the puddle as: $3 \times \frac{1}{3} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$	1
17	Finds the probability of a student being fluent in economics as: P(E) = 1 - P(M) = 1 - 0.75 = 0.25	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Represents mathematics as M and business studies as BS and uses Bayes' theorem to write: $$P(E\|BS) = \frac{P(E)P(BS\|E)}{P(E)P(BS\|E) + P(M)P(BS\|M)}$$ $$= \frac{0.25 \times 0.80}{(0.25 \times 0.80) + (0.75 \times 0.15)}$$ (Award 0.5 marks if only the formula for Bayes' theorem is written correctly.)	1
	Finds the probability of a student being fluent in economics given that he is fluent in business studies as $\frac{16}{25}$ or 64%.	0.5
18	Finds the P(it will not rain on Monday) as $1 - \frac{1}{6} = \frac{5}{6}$.	0.5
	Finds the P(it will not rain on Tuesday) as $1 - \frac{2}{5} = \frac{3}{5}$.	0.5
	Finds the P(it will not rain on Wednesday) as $1 - \frac{4}{5} = \frac{1}{5}$.	0.5
	Finds the P(it will not rain on all the three days) as $\frac{5}{6} \times \frac{3}{5} \times \frac{1}{5} = \frac{1}{10}$.	0.5
	Finds the P(it will rain on at least one of the three days) as: 1 - P(it will not rain on all the three days) $= 1 - \frac{1}{10} = \frac{9}{10}$ (Award 0.5 marks if only the first part of the step is correctly written.)	1

Q.No	Teacher should award marks if students have done the following:	Marks
19	Draws a tree diagram to illustrate the given situation. The diagram may look as follows: 	2
	Uses the above diagram and finds the probability of the flight being on time on Friday given that it was on time on Wednesday as $\left(\frac{1}{5} \times \frac{7}{10}\right) + \left(\frac{4}{5} \times \frac{4}{5}\right) = \frac{39}{50}$.	1
20	Writes the probability of Himansh having a misconception as P(M) = 0.6 and of not having a misconception as P(M') = 1 - 0.6 = 0.4.	0.5
	Writes the probability of the test outcome being green given he has a misconception as P(G|M) = 0.2.	0.5
	Writes the probability of the test outcome being green given he doesn't have a misconception as P(G|M') = 0.9.	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Uses Bayes' theorem and finds the probability that Himansh has a misconception given his test outcome was green as: $$P(M\|G) = \frac{P(M)P(G\|M)}{P(M)P(G\|M) + P(M')P(G\|M')}$$ $$= \frac{0.6 \times 0.2}{(0.6 \times 0.2) + (0.4 \times 0.9)}$$ **(Award 0.5 marks if only the formula for Bayes' theorem is written correctly.)**	1
	Evaluates the above expression to find the required probability as $\frac{1}{4}$ or 25%.	0.5
21	**Finds the probability of a black ball being in the sealed box as $P(B_1) = \frac{1}{2}$ and that of a white ball being in the sealed box as $P(B_1') = \frac{1}{2}$.**	1
	Finds the probability of picking a black ball, B_2 given that B_1 is black as: $P(B_2 \| B_1) = 1$	1
	Finds the probability of picking a black ball given that B_1 is white as: $P(B_2 \| B_1') = \frac{1}{2}$	1
	Finds the total probability of picking a black ball as: $P(B_2) = P(B_2\|B_1)P(B_1) + P(B_2\|B_1')P(B_1')$ $= \frac{1}{2} + (\frac{1}{2} \times \frac{1}{2})$ $= \frac{3}{4}$	1

Q.No	Teacher should award marks if students have done the following:	Marks
	Finds the probability that the original ball in the box is black given that he picked a black ball as: $$P(B_1 \mid B_2) = \frac{P(B_1)P(B_2 \mid B_1)}{P(B_2)} = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$$	1
22	Takes A to be an event of drawing a card with an even number on it and finds P(A) as $\frac{10+8+7+10+15+5}{100} = \frac{55}{100}$.	1
	Takes B to be an event of drawing a card with a multiple of 3 on it and finds P(B) as $\frac{12+7+12+5}{100} = \frac{36}{100}$.	1
	Writes that $A \cap B = \{6, 12\}$ and finds $P(A \cap B)$ as $\frac{7+5}{100} = \frac{12}{100}$.	0.5
	Finds the probability that the number on the card is even provided that it is a multiple of 3 as: $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$ $= \frac{12}{100} \div \frac{36}{100} = \frac{1}{3}$	1
	Finds the probability that the number on the card is a multiple of 3 provided that it is an even number as: $P(B \mid A) = \frac{P(A \cap B)}{P(A)}$ $= \frac{12}{100} \div \frac{55}{100} = \frac{12}{55}$	1
	Uses step 4 and 5 to write $P(A \mid B) \neq P(B \mid A)$ and hence concludes that what Danish said was wrong.	0.5
23	i) Takes A to be the event of picking an apple and finds $P(A) = \frac{4}{10} = \frac{2}{5}$.	0.5
	Finds the probability of getting an apple in the second pick with the condition that one apple has already been picked as $P(A \mid A) = \frac{3}{9} = \frac{1}{3}$.	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Finds the probability of getting an apple in the third pick with the condition that two apples have already been picked as P(A|AA) = $\frac{2}{8} = \frac{1}{4}$.	0.5
	Uses the multiplication theorem of probability and finds P(all three fruits picked are apples) as: P(AAA) = $\frac{2}{5} \times \frac{1}{3} \times \frac{1}{4} = \frac{1}{30}$ (Award 2 marks if the problem is solved correctly using combinatorics.)	0.5
	ii) Takes G to be the event of picking a guava and finds P(G) = $\frac{6}{10} = \frac{3}{5}$.	0.5
	Finds the probability of getting an apple in the second pick with the condition that one guava has already been picked as P(A|G) = $\frac{4}{9}$.	0.5
	Finds the probability of getting an apple in the third pick with the condition that one guava and one apple have already been picked as P(A|GA) = $\frac{3}{8}$.	0.5
	Uses the multiplication theorem of probability and finds P(the first fruit is guava and the next two are apples) as: P(GAA) = $\frac{3}{5} \times \frac{4}{9} \times \frac{3}{8} = \frac{1}{10}$ (Award 2 marks if the problem is solved correctly using combinatorics.)	0.5
	iii) Uses step 4 and finds P(at least one of the fruits picked is a guava) as: 1 - P(all three fruits picked are apples) $= 1 - \frac{1}{30}$ $= \frac{29}{30}$ (Award 0.5 marks if only the first part of the step is correctly written.)	1
24	Assumes the set of patients having fever and migraine to be F and M respectively. Finds the required probability as: P(M'|F) = 1 - P(M|F) $= 1 - \frac{P(M \cap F)}{P(F)}$ $= 1 - \frac{6}{18}$ $= \frac{2}{3}$ (Award 0.5 marks if just the formula is written correctly.)	2

Q.No	Teacher should award marks if students have done the following:	Marks
25	Assumes the set of patients having cold, fever and migraine to be C, F and M respectively. Writes that the required probability is given by, $P(C \cup F \mid M) = P(C \mid M) + P(F \mid M) - P(C \cap F \mid M)$	0.5
	Evaluates $P(C \mid M)$ as $\frac{P(C \cap M)}{P(M)} = \frac{3}{9}$.	0.5
	Evaluates $P(F \mid M)$ as $\frac{P(F \cap M)}{P(M)} = \frac{6}{9}$.	0.5
	Evaluates $P(C \cap F \mid M)$ as $\frac{P(C \cap F \cap M)}{P(M)} = \frac{2}{9}$.	0.5
	Finds the probability that a patient selected at random has cold or fever, provided he/she has migraine as: $\frac{3}{9} + \frac{6}{9} - \frac{2}{9} = \frac{7}{9}$ (Award 1.5 marks if the problem is solved correctly without explicitly writing the formula in step 1.)	1

Chapter - 14
Application of Multiple Concepts

Q: 1 **Find the critical point(s) of the real valued function, $f(x) = x^x$, where $x \in (0,\infty)$. Show your work.** **[2]**

Q: 2 **Prove that:** **[4]**

$$\sin^{-1}\frac{(\cos\theta - \sqrt{3}\sin\theta)(1+\tan^2\theta)}{(1+\sin^2\theta)(1+\tan^2\theta)+1} = \left(\frac{\pi}{6} - \theta\right)$$

Q.No	Teacher should award marks if students have done the following:	Marks
1	**Considers $y = x^x$ and takes logarithm to the base e on both sides to write:** **$\ln y = x \ln x$**	0.5
	Differentiates the above equation with respect to x as: $\frac{dy}{dx} = x^x(1 + \ln x)$	1
	Writes that, since x^x cannot be zero, $(1 + \ln x)$ must be zero. Solves this equation to get the critical point as: $1 + \ln x = 0$ $\Rightarrow x = e^{-1}$ or $\frac{1}{e}$ **(Award full marks if 'log' is used instead of 'ln'.)**	0.5
2	**Writes the L.H.S as:** $\sin^{-1}\left(\frac{(\cos\theta - \sqrt{3}\sin\theta)\sec^2\theta}{(1+\sin^2\theta)\sec^2\theta + 1}\right)$	0.5
	Rewrites the L.H.S as: $\sin^{-1}\left(\frac{(\cos\theta - \sqrt{3}\sin\theta)\sec^2\theta}{\sec^2\theta + \tan^2\theta + 1}\right)$	0.5
	Simplifies the L.H.S. as: $\sin^{-1}\left(\frac{(\cos\theta - \sqrt{3}\sin\theta)\sec^2\theta}{2\sec^2\theta}\right)$	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Simplifies the above step as: $\sin^{-1}\left(\frac{\cos\theta}{2} - \frac{\sqrt{3}\sin\theta}{2}\right)$	**0.5**
	Rewrites the above step as: $\sin^{-1}(\sin\frac{\pi}{6}\cos\theta - \sin\theta\ \cos\frac{\pi}{6})$	**1**
	Rewrites the above step as: $\sin^{-1}(\sin(\frac{\pi}{6} - \theta))$ **and proves that the L.H.S = R.H.S = ($\frac{\pi}{6}$ - θ).**	**1**

15. Annexure

Correct Answer Explanation

Chapter Name	Q. No	Correct Answer	Correct Answer Explanation
Relations and Functions	2	1	Since the number of elements in the codomain (set B) is greater than that in the domain (set A), for any function, there will be at least one element in the co-domain (set B) which is not an image of any element in the domain (set A). Hence option A is the correct answer.
	4	1	The given relation is reflexive as it contains all the elements of the form (x, x).
			The given relation is symmetric since for every (x, y) in the relation, (y, x) is also in the relation.
			The given relation is transitive as it DOES NOT contain any pair of elements of the form (x, y) and (y, z).
			Hence option A is the correct answer.
	7	2	$f(1)=1$, $f(2)=0$, $f(3)=2$, $f(4)=0$, $f(5)=3$... All the odd numbers are being mapped to positive integers and all the even numbers are being mapped to 0. The function is onto as all the non-negative integers of the co-domain are the images of some elements in the domain. The function is not one-one as more than one even number is getting mapped to 0. Hence, option B is the correct answer.
Inverse Trigonometric Functions	1	1	Domain of arcsec(x) is $(-\infty, -1] \cup [1, \infty)$ and domain of arcsin(x) is $[-1, 1]$. For the given function, the domain would be intersection of the two domains. That is, $(-\infty, -1] \cup [1, \infty) \cap [-1, 1] = \{-1, 1\}$. Hence, option A is the correct answer.
	3	3	Domain of arccos(x) is $[-1, 1]$. So $1/x-3$ should belong to $[-1, 1]$. Solving the inequality $-1 \leq 1/(x - 3) \leq 1$ gives $x \leq 2$ and $x \geq 4$. Hence, option C is the correct answer.
Matrices	1	4	i) Columns 1 and 3 are identical, so upon exchanging them, the matrix is unchanged. ii) Third row is a zero row, so adding any scalar multiple of it to another row will keep the row unchanged. iii) Column 2 is a zero column, so multiplying it with (-1) keeps the matrix unaltered. Hence option D is the correct answer.

	2	4	The order of the two matrices given are 2x3. A matrix of order 2x3 is not compatible for multiplication with another matrix of order 2x3. Only matrices with orders of the form 3xk, where k is a natural number, are compatible to multiply with matrices of order 2x3. Thus, none of their answers is correct.
			Hence, option D is the correct answer.
Determinants	5	3	Taking 3 common from the second row and 2 common from the third row one can observe that the elements of first row are identical to the elements of the third row. Given that the determinant is 0, this can happen when 2 rows are the same. So 2cos(2x) could come in the box which is the second element of row 3. Hence, option C is the correct answer.
Continuity and Differentiability	1	3	In g(x), (x-1) is differentiable everywhere. |x-1| is not differentiable at 1. Graph of g(x) shows that it is differentiable everywhere. Thus, product of a differentiable function and a non-differentiable function could be differentiable.
			In f(x), x is differentiable everywhere. |x-1| is not differentiable at 1. Graph of f(x) is pointed at x=1, that means it is not differentiable at 1. Thus, product of a differentiable function and a non-differentiable function could be non-differentiable.
			Therefore, product of a differentiable function and a non-differentiable function MAY BE differentiable. Hence, option C is the correct answer.
Application of Derivatives	4	2	f'(x)=2x^2-18. Equating f'(x) to zero gives the critical points as (+3) and (-3). f"(x)=4x. f"(-3)<0 and f"(3)>0, that means function attains maxima at (-3) and minima at (+3). Thus the function is decreasing in the interval [-3, 3]. Hence, option B is the correct answer.
Integrals	2	2	Statement 1 is false. For example, integral of (-x) in the interval (1, 2) is (-3/2). If integral exists in (a, b), then the function is continuous in (a, b), because, in an interval if the function is continuous only then it can be integrated. Thus, statement 2 is true.
			Hence, option B is the correct answer.
Differential Equations	5	4	The general solution of this differential equation is asinx + bcosx, where a and b are arbitrary constants. Bulbul's particular solution is obtained when a=1 and

			b=0. Ipsita's particular solution is obtained when a=0 and b=1. Sagarik's particular solution is obtained when a= 1 and b=1. Thus all three of their answers are particular solutions. Thus, option D is the correct answer.
Vector Algebra	2	3	v x u = -w. Angle between w and (-w) is 180 degrees. Hence, option C is the correct answer.
	3	2	Cross product of two vectors is given by |a||b|sin Θ, where Θ is the angle between a and b. |a| and |b| are positive being magnitudes. sin Θ is maximum when the angle between a and b is ninety degrees. Hence, option B is the correct answer.

Central Board of Secondary Education
Shiksha Sadan, 17, Rouse Avenue,
New Delhi-110002

Competency Focused Practice Questions

Mathematics (Volume 2) | Grade 12

Co-created by
CBSE Centre for Excellence in Assessment
and
Educational Initiatives

Preface

Assessments are an important tool that help gauge learning. They provide valuable feedback about the effectiveness of instructional methods; about what students have actually understood and also provide actionable insights. The National Education Policy, 2020 has outlined the importance of competency-based assessments in classrooms as a means to reform curriculum and pedagogical methodologies. The policy emphasizes on the development of higher order skills such as analysis, critical thinking and problem solving through classroom instructions and aligned assessments.

Central Board of Secondary Education (CBSE) has been collaborating with Educational Initiatives (Ei) in the area of assessment. Through resources like the Essential Concepts document and A- Question-A-Day (AQAD), high quality questions and concepts critical to learning have been shared with schools and teachers.

Continuing with the vision to ensure that every student is learning with understanding, Question Booklets have been created for subjects for Grade 10th and 12th. These booklets contain competency-based items, designed specifically to test conceptual understanding and application of concepts.

Process of creating competency-based items

All items in these booklets are aligned to the NCERT curriculum and have been created keeping in mind the learning outcomes that are important for students to understand and master. Items are a mix of Free Response Questions (FRQs) and Multiple-Choice Questions (MCQs). In case of MCQs, the options (correct answer and distractors) are specifically created to test for understanding and capturing specific errors/misconceptions that students may harbour. Each incorrect option can thereby inform teachers on specific gaps that may exist in student learning. In case of subjective questions, each question also has a detailed scoring rubric to guide evaluation of students' responses.

Each item has been reviewed by experts, to check for appropriateness of the item, validity of the item, conceptual correctness, language accuracy and other nuances.

How can these item booklets be used?

There are 137 questions in this booklet.

The purpose of these item booklets is to provide samples of high-quality competency-based items to teachers. The items can be used to–

- get an understanding of what good competency-based questions could look like
- give exposure to students to competency-based items
- assist in classroom teaching and learning
- get inspiration to create more such competency-based items

Students can also use this document to understand different kinds of questions and practice specific concepts and competencies. There will be further additions in the future to provide competency focused questions on all chapters.

The item booklets are aligned with the 2022-23 curriculum. However, a few questions from topic which got rationalized in 2023-24 syllabus are also there in the booklet which may be used as a reference for teachers and students.

Please write back to us to give your feedback.

Team CBSE

Table of Contents

Chapter - 1
Relations and Functions

Free Response Questions

Q: 1 **f is a strictly increasing function while g is a strictly decreasing function. The range of f and g are the same as the codomain of f and g respectively.** **[2]**

If (fog) is defined, will (fog) be an invertible function? Justify your answer.

Q: 2 ***Prathibha Karanji* is an innovative program by the Government of Karnataka, India, where cultural and literacy competitions are held between schools at cluster, block, district and state levels.** **[5]**

One of those competitions - Yogasana, is conducted under two categories - Middle school and High school. From a certain district, three students from middle school and two students from high school were selected for the state level.

Let M = { m_1, m_2, m_3 }, H = { h_1, h_2 }, represent the set of students from middle school and high school respectively who got selected for the state level from that district.

i) A relation R:M -> M is defined by R = {(x, y): x and y are students from the same category}. Show that R is an equivalence relation.

ii) A function f :M -> H is defined by f = {(m_1, h_1), (m_2, h_2), (m_3, h_2)} . Show that f is onto but not one-one.

Q: 3 **Two functions f : R -> R and g : R -> R, where R is the set of real numbers, are defined as follows:** **[5]**

f (x) = (x + sin x)
g (x) = (- x - cos x)

i) Find (fog)(x).
ii) Find (gof)(x).
iii) Using i) and ii), show that (gof)(0) - sin(1) = (fog)(0).

Show your steps.

Q: 4 **[5]**

If $f(x) = 3 + \left(\frac{e^{3x}+e^{-3x}}{e^{3x}-e^{-3x}}\right)$ and $f^{-1}(x) = \frac{1}{A}g(x)$, find :

i) the value of A.
ii) $g(x)$.

Show your steps.

Q.No	What to look for	Marks
1	Writes that, *f* and *g* are one-to-one functions since they are strictly increasing and strictly decreasing respectively.	0.5
	Writes that, (*fog*) is also one-to-one since *f* and *g* are one-to-one.	0.5
	Writes that, (*fog*) is onto since it is given that *f* and *g* are onto functions.	0.5
	Concludes that, (*fog*) is a bijective function and therefore invertible.	0.5
2	i) Shows that R is reflective. The working may look as follows: x and x are from the same category => $(x, x) \in$ R for every $x \in$ M	0.5
	Shows that R is symmetric. The working may look as follows: $(x, y) \in$ R => x and y are from the same category. => y and x are from the same category. => $(y, x) \in$ R	1
	Shows that R is transitive. The working may look as follows: $(x, y) \in$ R and $(y, z) \in$ R => x and y are from the same category and, y and z are from the same category. => x, y and z are from the same category. => $(x, z) \in$ R	1
	Uses steps 1, 2 and 3 to conclude that R is an equivalence relation.	0.5
	ii) Writes that the Range of $f = \{h_1, h_2\}$ = codomain of f. Hence concludes that f is onto.	1

Q.No	What to look for	Marks
	Writes that $f(m_2) = f(m_3) = h_2$, but $m_2 \neq m_3$. Hence concludes that f is not one-one.	**1**
3	**i) Finds (*fog*)(*x*) as follows:** $f[g(x)] = f(-x - \cos x) = -x - \cos x + \sin(-x - \cos x)$	**1.5**
	ii) Finds (*gof*)(*x*) as follows: $g[f(x)] = g(x + \sin x) = -(x + \sin x) - \cos(x + \sin x)$	**1.5**
	iii) Substitutes $x = 0$ in the expression obtained in step 1 to find (*fog*)(0) as [-1 - sin(1)].	**0.5**
	Substitutes $x = 0$ in the expression obtained in step 2 to find (*gof*)(0) as (-1).	**0.5**
	Finds (*gof*)(0) - sin(1) as [-1 - sin(1)]. **Concludes that (*gof*)(0) - sin(1) = (*fog*)(0).**	**1**
4	**Equates f(x) to y and writes:** $f(x) = y = 3 + \left(\frac{e^{6x}+1}{e^{6x}-1}\right)$	**0.5**
	Simplifies the above equation to get: $y = \frac{4e^{6x}-2}{e^{6x}-1}$	**1**
	Simplifies the above equation to get: $e^{6x} = \frac{y-2}{y-4}$	**1**

Q.No	What to look for	Marks
	Simplifies the above equation to get: $x = f^{-1}(y) = \frac{1}{6} \log_e \frac{y-2}{y-4}$	**1.5**
	Rewrites the above equation in terms of *x* as: $f^{-1}(x) = \frac{1}{6} \log_e \frac{x-2}{x-4}$	**0.5**
	Compares the above equation with the given $f^{-1}(x)$ and writes: $A = 6 \text{ and } g(x) = \log_e \left(\frac{x-2}{x-4}\right)$	**0.5**

Chapter - 2
Inverse Trigonometric Functions

Multiple Choice Questions

Q: 1 **Which of the following is equal to $-\tan^{-1}\left(\frac{2}{3\pi}\right)$?**

1. $\cot^{-1}\left(\frac{3\pi}{2}\right)$
2. $-\cot^{-1}\left(\frac{3\pi}{2}\right)$
3. $\frac{\pi}{2} - \cot^{-1}\left(\frac{3\pi}{2}\right)$
4. $\frac{\pi}{2} + \cot^{-1}\left(\frac{3\pi}{2}\right)$

Free Response Questions

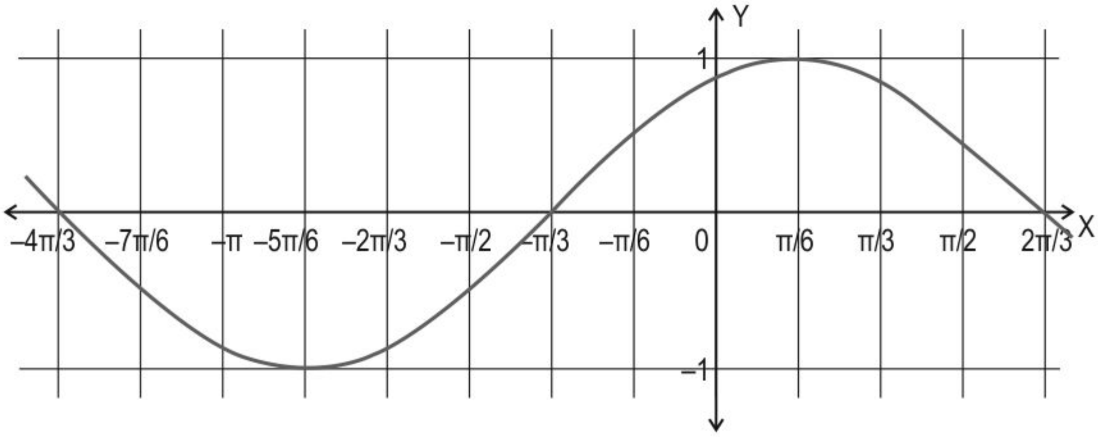
Q: 2 **Draw the graph of $\cos^{-1}(2x)$ in the domain $[-\frac{1}{2}, \frac{1}{2}]$.** **[1]**

Q: 3 **State whether the following statements are true or false. Explain your reasoning.** **[2]**

i) $\cos(\sec(x))^{-1} = \frac{1}{x}$

ii) $\sin^{-1}(-x) + \cos^{-1}(-x) = \sin^{-1}(x) - \cos^{-1}(x)$

Q: 4 **Shown below is a portion of the graph for $\sin(x + a)$, where $a \in R$ and $a > 0$.** **[2]**



What should the domain of the function be restricted to such that the function is invertible? Give valid reasons for your answer.

Q: 5 **If $4x + \frac{1}{x} = 4$, $x \in [0, \frac{\pi}{2}]$, find $\cos^{-1}x - \sin^{-1}x$. Show your work.** **[2]**

Q: 6 Show that $\cot^{-1}\left(\frac{1-x^2}{2x}\right) + \tan^{-1}\left(\frac{x^3-3x}{1-3x^2}\right) = -\tan^{-1}x$. **[3]**

Q: 7 **$f(x) = \cos(2\sin^{-1} x)$ and $g(x) = \sin^2(2\cos^{-1} x)$, $-1 \le x \le 1$.** **[3]**

If $h(x) = (f(x) + g(x))$, find $h(0.1)$. Show your work.

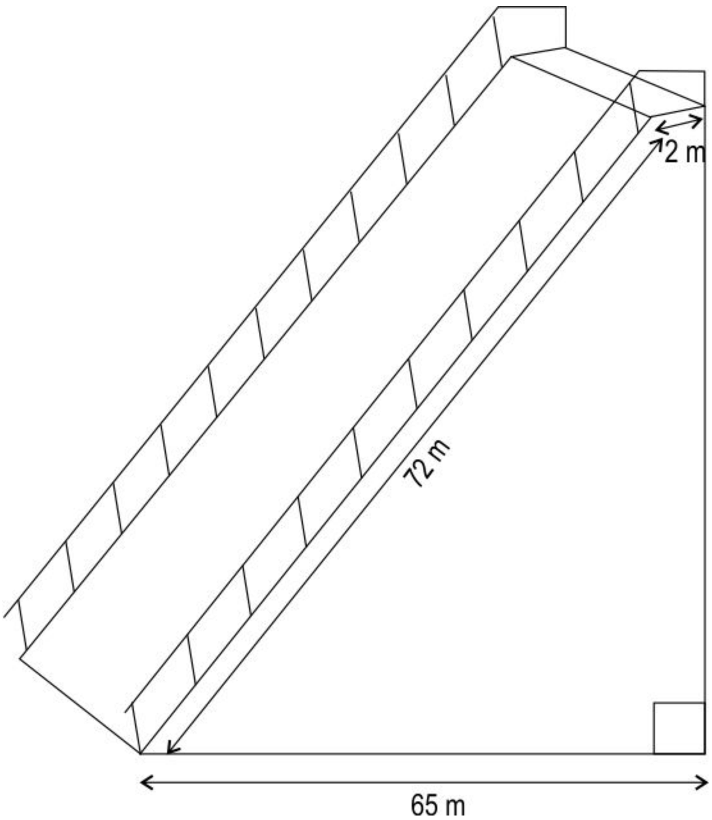
Q: 8 $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$ **[3]**

For $y \in \mathrm{R}$ where $\operatorname{cosec}^{-1} y$ and $\sin^{-1} y$ are defined,
is $\operatorname{cosec}^{-1} y = \frac{1}{\sin^{-1} y}$? If not, write the correct statement.

Q: 9 **Find the domain and range of the function $y = \sin^{-1}(|x| - 1)$. Show your work and give valid reasons.** **[3]**

Q: 10 **As per guidelines issued by the Home Ministry of India, in order for a ramp to be accessible to persons with disability, it must have a minimum slope ratio of 1:12. That is, for every unit of height, there must be at least 12 units of ramp.** **[3]**
(Source: https://www.mha.gov.in/sites/default/files/PublicNoticeforDraftStandards_22112021.pdf)

Abbas measured the dimensions of a ramp at his school and noted them in a rough diagram, as shown in the figure below.



(Note: The figure is not to scale.)

Is the given ramp meeting the Home Ministry's accessibility standards? Justify your answer.

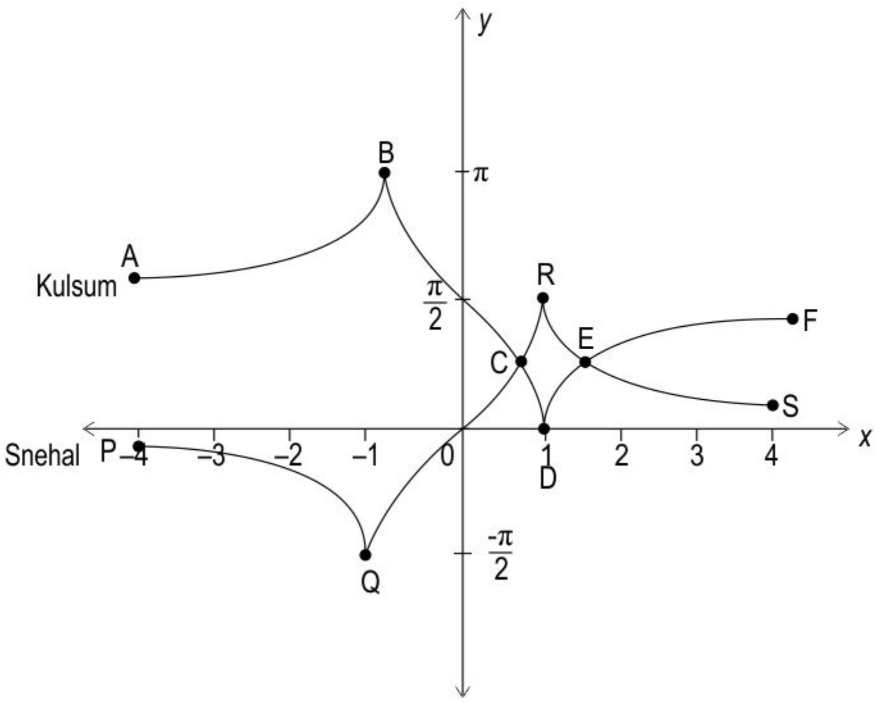
(Note: Assume that the top of the ramp is parallel to the ground.)

Q: 11 Evaluate $\tan\left(\frac{1}{2}\cos^{-1}\frac{5}{7}\right)$. Show your work. **[5]**

Case Study

Answer the questions based on the given information.
Madhu created a design on his floor using a combination of graphs of inverse trigonometric functions in the domain [-4, 4]. He also represented the coordinate axes for reference. He asked his friends, Kulsum and Snehal, to choose a path to walk on. Kulsum chose path ABCDEF, while

Snehal chose path PQCRES. They both started walking at the same time and with the same speed.



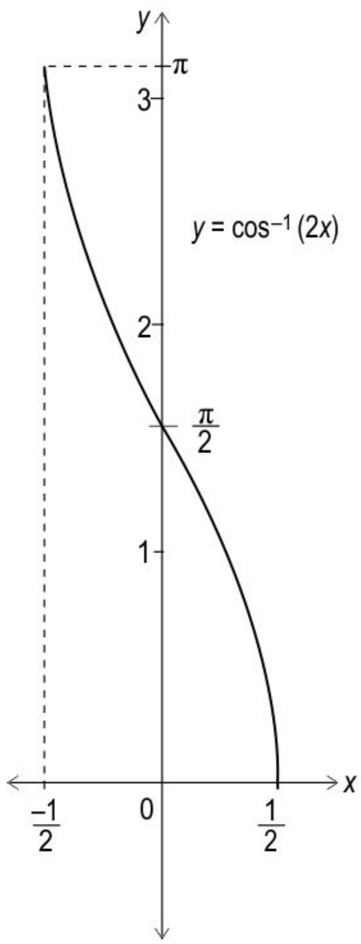
Q: 12 **Write the range of each of the two functions that Kulsum chose as her path to walk on.** **[2]**

Q: 13 **The graphs of which of the two functions are combined to form the path that Snehal chose?** **[1]**

Q: 14 **What is the *x* -coordinate of Kulsum's and Snehal's first meeting point? Show your steps.** **[2]**

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	2

Q.No	What to look for	Marks
2	Draws the graph of $\cos^{-1}(2x)$ in the given domain as follows: 	1
3	i) Writes that the given statement is false.	0.5
	Gives reason that $(\sec(x))^{-1} = \frac{1}{\sec x}$ and not $\sec^{-1}(x)$.	0.5
	ii) Writes that the given statement is false.	0.5

Q.No	What to look for	Marks
	Gives reason that $\sin^{-1}(-x) = -\sin^{-1}(x)$, and $\cos^{-1}(-x) = \cos^{-1}(x)$. So, $\sin^{-1}(-x) + \cos^{-1}(-x) = \cos^{-1}(x) - \sin^{-1}(x)$.	0.5
4	Writes that the domain should be restricted to $[\frac{-5\pi}{6}, \frac{\pi}{6}]$. (Award full marks for any domain that satisfies the conditions.)	1
	Reasons that if its domain is restricted to $[\frac{-5\pi}{6}, \frac{\pi}{6}]$, then the function becomes one-one and onto and hence, invertible.	1
5	Rewrites $4x + \frac{1}{x} = 4$ as $(2x - 1)^2 = 0$. Hence, finds the value of x as $\frac{1}{2}$.	1
	Evaluates $\cos^{-1} x - \sin^{-1} x$ as $\frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$ or 30°.	1
6	Writes that: $\cot^{-1}\left(\frac{1-x^2}{2x}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$	0.5
	Takes $x = \tan\theta$. Then $\theta = \tan^{-1} x$. Solves the LHS by substituting the expression from step 1 as follows: $\tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right) + \tan^{-1}\left(\frac{\tan^3\theta-3\tan\theta}{1-3\tan^2\theta}\right)$ $= \tan^{-1}(\tan 2\theta) + \tan^{-1}(-\tan 3\theta)$	1.5
	Finds $\tan^{-1}(-\tan(3\theta)) = \tan^{-1}(\tan(-3\theta))$.	0.5
	Simplifies the expression from step 2 as $\tan^{-1}(\tan(2\theta)) + \tan^{-1}(\tan(-3\theta))$ $= 2\theta - 3\theta = -\theta = -\tan^{-1} x$.	0.5

Q.No	What to look for	Marks
7	**Using the formula $\cos 2\theta = 1 - 2\sin^2 2\theta$, writes $f(x)$ as follows:** $\cos(2\sin^{-1} x) = 1 - 2\sin^2(\sin^{-1} x) = 1 - 2x^2$	1
	Using the formula $\sin 2\theta = 2\sin\theta.\cos\theta$, writes $g(x)$ as follows: $\sin^2(2\cos^{-1} x) = 4\sin^2(\cos^{-1} x).\cos^2(\cos^{-1} x)$ $= 4\cos^2(\cos^{-1} x)(1 - \cos^2(\cos^{-1} x))$ $= 4x^2(1 - x^2) = 4x^2 - 4x^4$	1
	Finds $h(x) = (1 - 2x^2) + (4x^2 - 4x^4) = 1 + 2x^2 - 4x^4$	0.5
	Calculates $h(0.1)$ as $1 + 2(0.1)^2 - 4(0.1)^4 = 1.0204$	0.5
8	Writes: $\text{cosec}^{-1} y = x$ $\Rightarrow \text{cosec}\, x = y$ $\Rightarrow \frac{1}{\sin x} = y$ $\Rightarrow \sin x = \frac{1}{y}$ $\Rightarrow \sin^{-1} \frac{1}{y} = x = \text{cosec}^{-1} y$	1.5
	Uses the given relation and the above step to write: $\sin^{-1} \frac{1}{y} = \frac{1}{\sin^{-1} y}$ which is not true for $y = 1$. Hence, $\text{cosec}^{-1} y = \frac{1}{\sin^{-1} y}$ is not true.	1.5
9	**Since the domain of inverse of sine function is [-1, 1], finds the domain of the given function as follows:** $-1 \le \lvert x \rvert - 1 \le 1$ $\Rightarrow 0 \le \lvert x \rvert \le 2$ $\Rightarrow \lvert x \rvert \le 2$ $\Rightarrow -2 \le x \le 2$	1.5

Q.No	What to look for	Marks
	Concludes the domain of $\sin^{-1}(\lvert x \rvert - 1)$ as [-2, 2].	**0.5**
	Finds the range of the function as an interval of real numbers where sin *y* is bijective. **Some examples are:** $[-\frac{\pi}{2}, \frac{\pi}{2}]$, $[-\frac{3\pi}{2}, -\frac{\pi}{2}]$, ...	**0.5**
	Gives a valid reason. For example, since sin *y* is bijective in the domain $[-\frac{\pi}{2}, \frac{\pi}{2}]$, the range of the given function is $[-\frac{\pi}{2}, \frac{\pi}{2}]$.	**0.5**
10	**Assumes the angle of inclination of the given ramp as θ.** **Finds the base length of the given ramp as 63 m, and hence finds θ as $\cos^{-1}\frac{63}{72} = \cos^{-1}\frac{7}{8}$.**	**0.5**
	Notes that, for a ramp to meet the given accessibility standards, the angle of inclination must be less than or equal to $\sin^{-1}\frac{1}{12}$.	**0.5**
	Writes $\sin^{-1}\frac{1}{12}$ as follows: $\sin^{-1}\left(\frac{1}{12}\right) = \cos^{-1}\left(\sqrt{1-\left(\frac{1}{12}\right)^2}\right)$ $\Rightarrow \sin^{-1}\left(\frac{1}{12}\right) = \cos^{-1}\left(\frac{\sqrt{143}}{12}\right)$	**1**
	Notes that : $\cos^{-1}\left(\frac{7}{8}\right) > \cos^{-1}\left(\frac{\sqrt{143}}{12}\right)$ **Gives a valid reason. For example, the value of cos θ decreases as θ increases, when 0° ≤ θ ≤ 90°.** **Concludes that the ramp is not meeting the accessibility standards set by the Home Ministry.** **(Award no marks if student simply writes that the ramp does not meet the accessibility standards, if no valid reason is given.)**	**1**

Q.No	What to look for	Marks
11	Assumes that $\frac{1}{2}\cos^{-1}\frac{5}{7} = \theta$. Then, $\cos^{-1}\frac{5}{7} = 2\theta$ $\Rightarrow \cos 2\theta = \frac{5}{7}$	0.5
	Writes cos 2θ as follows: $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{5}{7}$	1
	Solves the equation from step 3 to get: $\tan^2\theta = \frac{1}{6}$ $\Rightarrow \tan\theta = \left(\sqrt{\frac{1}{6}}\right)$ or $\left(-\sqrt{\frac{1}{6}}\right)$	1
	Writes that $0 \le \cos^{-1}\frac{5}{7} \le \pi$. $\Rightarrow 0 \le \frac{1}{2}\cos^{-1}\frac{5}{7} \le \frac{\pi}{2}$ $\Rightarrow 0 \le \tan\theta \le \infty$	2
	Rejects the negative value for tan θ, and writes that: $\tan\theta = \frac{1}{\sqrt{6}}$	0.5
12	**Identifies two functions that Kulsum chose to walk on as $\cos^{-1}(x)$ and $\sec^{-1}(x)$.**	1
	Writes that the range of $\cos^{-1}(x)$ is $[0, \pi]$.	0.5
	Writes that the range of $\sec^{-1}(x)$ is $[0, \pi] - \frac{\pi}{2}$.	0.5
13	**Identifies the trigonometric functions that Snehal chose to walk on as $\sin^{-1}(x)$ and $\text{cosec}^{-1}(x)$.**	1

Q.No	What to look for	Marks
14	**Mentions that both friends met at point C which is the point of intersection of two functions, $\sin^{-1}(x)$ and $\cos^{-1}(x)$ and writes:** $\cos^{-1}(x) = \sin^{-1}(x)$	0.5
	Simplifies the above equation as: $\cos(\cos^{-1}(x)) = \cos(\sin^{-1}(x))$ $\Rightarrow x = \cos(\sin^{-1}(x))$ **Substitutes $\sin^{-1}(x)$ as u and simplifies the above equation as:** $\sin u = \cos u$ $\Rightarrow \sin u = \sin(\frac{\pi}{2} - u)$ $\Rightarrow u = \frac{\pi}{4}$ **(Award full marks if $\cos^{-1}(x)$ and $\sin^{-1}(x)$ are shown to be equal at $y = \frac{\pi}{4}$ directly.)**	1
	Finds the x-coordinate of Kulsum's and Snehal's first meeting point as: $x = \sin u = \sin(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$	0.5

Chapter - 3
Matrices

Multiple Choice Questions

Q: 1 $M = \begin{bmatrix} 2 & 3 \\ 6 & 4 \end{bmatrix}$

Given below are two statements in relation with the above matrix- one labelled Assertion (A) and the other labelled Reason (R). Read the statements carefully and choose the option that correctly describes statements (A) and (R).

Assertion *(A)* : The matrix M cannot be expressed as the sum of a symmetric and a skew-symmetric matrix.

Reason *(R)* : The matrix M is neither a symmetric matrix nor a skew-symmetric matrix.

1. Both (A) and (R) are true and (R) is the correct explanation for (A).
2. Both (A) and (R) are true but (R) is not the correct explanation for (A).
3. (A) is false but (R) is true.
4. Both (A) and (R) are false.

Free Response Questions

Q: 2 **The following are two non-zero matrices M and N:** **[1]**

$$\mathrm{M} = \begin{bmatrix} 0 & a \\ 0 & b \end{bmatrix} \quad \mathrm{N} = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$$

What is the necessary condition for their product to be a zero matrix? Show your work and give valid reason.

Q: 3 **Shown below is matrix P.** **[3]**

$$\mathrm{P} = \begin{bmatrix} 3 & -1 \\ 1 & -2 \end{bmatrix}$$

Find one such matrix Q such that (P + Q) is a skew-symmetric matrix. Show your work.

Q: 4 **Drona is solving a bank robbery. He has a note from one of his informants, Arjun, which contains the robber's name in the form of an encoded matrix N.** **[5]**

♦ The decoded name is stored in the form of a matrix M with numbers representing letters, such that 1 = A, 2 = B, 3 = C, and so on until 26 = Z.
♦ The entries are to be read column-wise.
♦ Arjun's notes to Drona are encoded using an encoder matrix, E.
♦ Matrix E is multiplied with Matrix M, to get the matrix in the note, N.

Matrices N and E are given below.

$$\begin{bmatrix} 62 & 52 \\ 25 & 21 \\ -51 & -51 \end{bmatrix} \quad \Bigg| \quad \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ -1 & -2 & -2 \end{bmatrix}$$

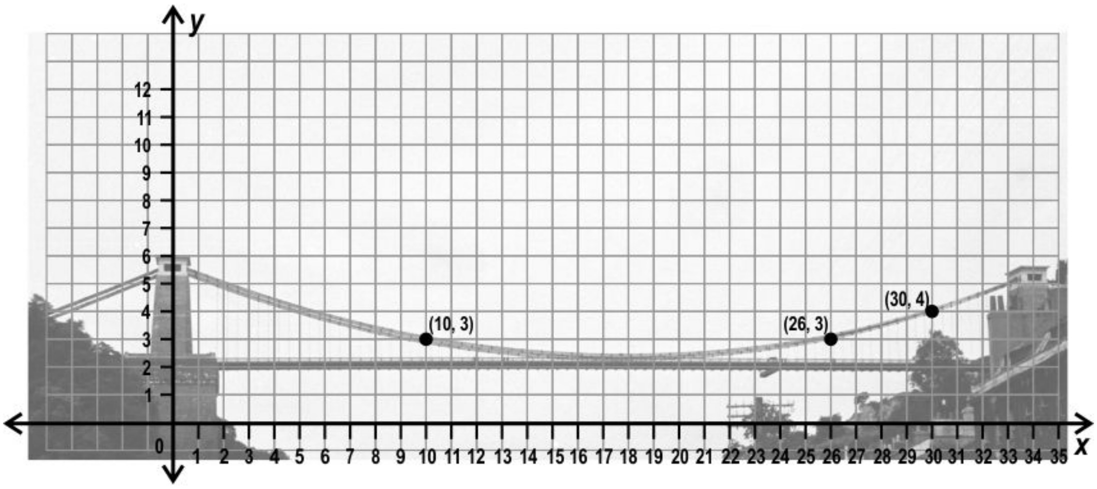
Matrix the note (N) | Encoder Matrix (E)

Find the key to decode the matrix N, and use it to find the robber's name. Show your work.

(Note: Encoding a matrix means to convert it to a coded form; Decoding a matrix means to convert it from coded form to normal form.)

Q: 5 **As part of an architecture project, Henry needs to create a miniature model of the Clifton Bridge, located in Bristol, UK. In order to do this, he must first find the equation of a parabola equivalent to the one made in Clifton Bridge.** **[5]**

He found a picture of the Clifton bridge, and placed it on a coordinate grid, as shown below. He noted that the parabola crossed the points (10, 3), (26, 3) and (30, 4) on the grid.



If the required parabola is of the form $y = ax^2 + bx + c$, then help Henry by finding *a*, *b* and *c*, and hence finding the equation of the parabola. Show your work.

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	3

Q.No	What to look for	Marks
2	**Writes the product of the matrices M and N as follows:** $MN = \begin{bmatrix} ar & as \\ br & bs \end{bmatrix}$	0.5
	Writes that the necessary condition for MN to be a zero matrix is that the values of *r* and *s* must always be 0 because *a* and *b* cannot be zero as M is a non-zero matrix.	0.5
3	**Represents the matrix (P + Q) as shown below:** $P + Q = \begin{bmatrix} a+3 & b-1 \\ c+1 & d-2 \end{bmatrix}$ where $Q = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$	0.5
	Uses the skew-symmetric relation (P + Q) = -(P + Q)' to write the equation as: $\begin{bmatrix} a+3 & b-1 \\ c+1 & d-2 \end{bmatrix} = \begin{bmatrix} -(a+3) & -(c+1) \\ 1-b & 2-d \end{bmatrix}$	1
	Finds the value of *a* as (-3), *d* as 2 and the relation *b* = - *c* using the equality of matrices in the above step and solve the following equations: ***a* + 3 = -(*a* + 3)** ***d* - 2 = 2 - *d*** **and** ***b* - 1 = -(*c* + 1)**	1

Q.No	What to look for	Marks
	Finds the matrix Q as shown below where the numerical values of *b* and *c* are such that *c* = - *b* : $$Q = \begin{bmatrix} -3 & b \\ -b & 2 \end{bmatrix}$$	**0.5**
4	**Notes that the determinant of Matrix E ≠ 0.** **Writes that, since EM = N, we can multiply both sides by E^{-1} to get E^{-1} EM = E^{-1} N. Simplifies the above expression to get M = E^{-1} N.**	**1**

Q.No	What to look for	Marks
	Finds the inverse of matrix E using elementary row operations. **The solution may look as follows:** $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ -1 & -2 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} E$ $R_3 \to R_3 + R_1$ $\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} E$ $R_1 \to R_1 - 2R_2 - R_3$ $\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -2 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} E$ $R_2 \to R_2 - R_3$ $\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -2 & -1 \\ -1 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix} E$ $\Rightarrow E^{-1} = \begin{bmatrix} 0 & -2 & -1 \\ -1 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix}$ **(Award full marks if inverse is found using adj(E) ÷ det(E).)**	2

Q.No	What to look for	Marks
	Multiplies E^{-1} with N as follows to get M: $\begin{bmatrix} 0 & -2 & -1 \\ -1 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix} . \begin{bmatrix} 62 & 52 \\ 25 & 21 \\ -51 & -51 \end{bmatrix}$ $= \begin{bmatrix} 0 + (-50) + 51 & 0 + (-42) + 51 \\ (-62) + 25 + 51 & (-52) + 21 + 51 \\ 62 + 0 + (-51) & 52 + 0 + (-51) \end{bmatrix}$ $= \begin{bmatrix} 1 & 9 \\ 14 & 20 \\ 11 & 1 \end{bmatrix}$	1.5
	Decodes the letters representing the numbers and reads them column-wise to get the robber's name as ANKITA.	0.5
5	**Writes the system of equations as:** **100 *a* + 10 *b* + *c* = 3** **676 *a* + 26 *b* + *c* = 3** **900 *a* + 30 *b* + *c* = 4**	0.5
	Writes the above system of equations in the form AX = B as follows: $\begin{pmatrix} 100 & 10 & 1 \\ 676 & 26 & 1 \\ 900 & 30 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix}$	0.5
	Finds |A| as 1(20280 - 23400) - 1(3000 - 9000) + 1(2600 - 6760) = -1280. **Writes that A^{-1} exists as |A| ≠ 0.**	0.5
	Finds A^{-1} as: $A^{-1} = \begin{pmatrix} \frac{1}{320} & -\frac{1}{64} & \frac{1}{80} \\ -\frac{7}{40} & \frac{5}{8} & -\frac{9}{20} \\ \frac{39}{16} & -\frac{75}{16} & \frac{13}{4} \end{pmatrix}$ **(Award 1 mark if only all the cofactors are found correctly.)**	2

Q.No	What to look for	Marks
	Finds the values of *a*, *b* and *c* as $\frac{1}{80}$, $-\frac{9}{20}$ and $\frac{25}{4}$ respectively by solving for matrix X = $A^{-1}B$ as follows: $$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} \frac{1}{320} & -\frac{1}{64} & \frac{1}{80} \\ -\frac{7}{40} & \frac{5}{8} & -\frac{9}{20} \\ \frac{39}{16} & -\frac{75}{16} & \frac{13}{4} \end{pmatrix} \times \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} \frac{1}{80} \\ -\frac{9}{20} \\ \frac{25}{4} \end{pmatrix}$$	**1**
	Writes that the equation of a parabola equivalent to that of the Clifton bridge is $\frac{1}{80}x^2 - \frac{9}{20}x + \frac{25}{4}$.	**0.5**

Chapter - 4
Determinants

Multiple Choice Questions

Q: 1 **U and V are two non-singular matrices of order *n* and *k* is any scalar.**

Given below are two statements based on the above information - one labelled Assertion (A) and the other labelled Reason (R). Read the statements carefully and choose the option that correctly describes statements (A) and (R).

Assertion (A) **: det(*k* U) = k^n × det(U)**

Reason (R) **: If W is a matrix obtained by multiplying any one row or column of V by the scalar *k*, then det(W) = *k* × det(V).**

1. Both (A) and (R) are true and (R) is the correct explanation for (A).
2. Both (A) and (R) are true but (R) is not the correct explanation for (A).
3. (A) is true but (R) is false.
4. (A) is false but (R) is true.

Free Response Questions

Q: 2 [1]

Show that for any positive integer n, $\begin{vmatrix} (n-1)! & n! \\ -n & n \end{vmatrix} = (n+1)!$.

(Note: n! denotes factorial n.)

Q: 3 **Given below is a system of linear equations in three variables.** [2]

$tx + 3y - 2z = 1$
$x + 2y + z = 2$
$-tx + y + 2z = -1$

For what value of *t*, will the system fail to have a unique solution? Show your work.

Q: 4 **P(*k*, -1), Q(3, -5) and R(6, -1) are vertices of ΔPQR. The lengths of side QR and altitude PS are 5 units and 4 units respectively.** [2]

Use determinants to find the value(s) of *k*. Show your work.

Q: 5 [2]

If $A = \begin{pmatrix} 4 & -8 \\ 12 & 16 \end{pmatrix}$ and $B = \begin{pmatrix} -16 & -8 \\ 12 & -4 \end{pmatrix}$,

identify what type of matrix is (Adj A) × (Adj B).

Show your work.

Q: 6 **[3]**

Consider the matrix $X = \begin{bmatrix} p & -2 & -3 \\ -2 & 2 & 6 \\ 1 & 3 & q \end{bmatrix}$, where p and $q \in$ R.

The co-factor of element 6 is (-11) and the minor of element 2 is 0.

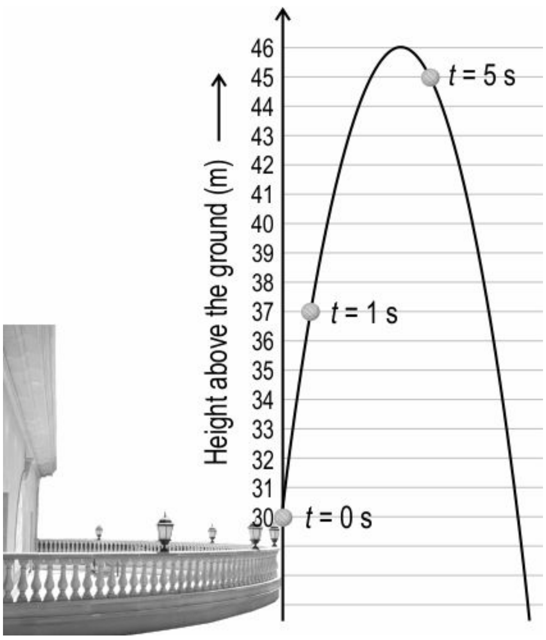
Find the values of *p* and *q*. Show your work.

Q: 7 **Without expanding the determinant, show that Δ is a perfect square for any integer values of *a*, *b* and *c*.** **[5]**

$$\Delta = \begin{vmatrix} b^2 + c^2 & a^2 & a^2 \\ b^2 & c^2 + a^2 & b^2 \\ c^2 & c^2 & a^2 + b^2 \end{vmatrix}$$

Q: 8 **A ball is thrown from a balcony. Its height above the ground after *t* seconds is given by $h(t) = pt^2 + qt + r$, where p, q, and $r \in$ R, and $h(t)$ is in meters.** **[5]**

Shown below is the trajectory of the ball with its height from the ground at *t* = 0 s, *t* = 1 s and *t* = 5 s.



Solve for *p*, *q* and *r* and find *h(t)* using the matrix method. Show your work and give valid reasons.

Q: 9 **A and B are invertible matrices of same order such that:** **[5]**

$$(AB)^{-1} = \frac{1}{48}\begin{bmatrix} 0 & 0 & 16 \\ 0 & 24 & 0 \\ 3 & 0 & 0 \end{bmatrix}$$

$$\text{and } A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Find B. Show your work.

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	1

Q.No	What to look for	Marks
2	**Expands the determinant as:** $n(n-1)! + n(n!)$	0.5
	Simplifies the above expression as: $n! + n(n!) = n!(n+1) = (n+1)!$	0.5
3	**Writes the coefficient matrix and finds its determinant as:** $\begin{vmatrix} t & 3 & -2 \\ 1 & 2 & 1 \\ -t & 1 & 2 \end{vmatrix} = t(3) - 3(2+t) - 2(1+2t) = -4t - 8$	1
	Solves the equation -4 t - 8 = 0 for t and writes that for t = (-2), the system fails to have a unique solution.	1
4	**Uses QR and PS to find the area of the triangle as 10 sq units. The working may look as follows:** $\frac{1}{2} \times 5 \times 4 = 10$	0.5
	Equates the area found to that using determinants as follows: $\frac{1}{2}\begin{vmatrix} k & -1 & 1 \\ 3 & -5 & 1 \\ 6 & -1 & 1 \end{vmatrix} = 10$	0.5
	Expands the above determinant to write the equation as: $\frac{1}{2}\left\| k(-5+1) + 1(3-6) + 1(-3+30) \right\| = 10$ $\left\| -4k + 24 \right\| = 20$	0.5

Q.No	What to look for	Marks
	Solves the above equation to find the values of *k* as 1 or 11.	**0.5**
5	**Finds (Adj A) as:** $\begin{pmatrix} 16 & 8 \\ -12 & 4 \end{pmatrix}$	**0.5**
	Finds (Adj B) as: $\begin{pmatrix} -4 & 8 \\ -12 & -16 \end{pmatrix}$	**0.5**
	Finds the product of (Adj A) and (Adj B) as: $\begin{pmatrix} -160 & 0 \\ 0 & -160 \end{pmatrix}$	**0.5**
	Identifies the matrix (Adj A) × (Adj B) as a scalar matrix. **(Award full marks if diagonal matrix is written instead of scalar matrix.)**	**0.5**
6	**Frames an equation in *p* using the co-factor of element 6 as** $A_{23} = (-1)^{2+3} \begin{vmatrix} p & -2 \\ 1 & 3 \end{vmatrix} = -3p - 2 = -11$	**1**
	Solves the above equation to find the value of *p* as 3.	**0.5**
	Frames an equation in *q* using the minor of element 3 as $M_{22} = \begin{vmatrix} p & -3 \\ 1 & q \end{vmatrix} = \begin{vmatrix} 3 & -3 \\ 1 & q \end{vmatrix} = 3p + 3 = 0$	**1**
	Solves the above equation to find the value of *q* as (-1).	**0.5**

Q.No	What to look for	Marks
7	**Performs the row operation $R_1 \to R_1 - R_2 - R_3$ on ∆ to get:** $$\Delta = \begin{vmatrix} 0 & -2c^2 & -2b^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$	1
	Takes (-2) common from the first row and performs the row operation $R_3 \to R_3 - R_2$ on ∆ to get: $$\Delta = (-2)\begin{vmatrix} 0 & c^2 & b^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2-b^2 & -a^2 & a^2 \end{vmatrix}$$	1
	Performs the row operation $R_2 \to R_2 - R_1$ on ∆ to get: $$\Delta = (-2)\begin{vmatrix} 0 & c^2 & b^2 \\ b^2 & a^2 & 0 \\ c^2-b^2 & -a^2 & a^2 \end{vmatrix}$$	1
	Performs the row operation $R_3 \to R_3 + R_2$ on ∆ to get: $$\Delta = (-2)\begin{vmatrix} 0 & c^2 & b^2 \\ b^2 & a^2 & 0 \\ c^2 & 0 & a^2 \end{vmatrix}$$	1

Q.No	What to look for	Marks
	Expands the determinant to get: $\Delta = (-2)[-c^2(b^2a^2) + b^2(-c^2a^2)]$ $= 4a^2b^2c^2$ $= (2abc)^2$ **Hence concludes that Δ is a perfect square for any any integer value of a, b and c.**	**1**
8	**Finds $r = 30$ by substituting $t = 0$ in $h(t)$.**	**0.5**
	Frames the following equations in p and q by substituting $t = 1$ and $t = 5$ respectively: $p + q = 7$ $5p + q = 3$	**0.5**
	Writes the above system of equations in the matrix form using AX = B as: $\begin{bmatrix} 1 & 1 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$	**0.5**
	Finds $\lvert A \rvert$ as -4 ($\neq 0$). Hence, writes that A^{-1} exists and the system has a unique solution.	**0.5**
	Finds *adj* A as: $\begin{bmatrix} 1 & -1 \\ -5 & 1 \end{bmatrix}$	**0.5**
	Finds A^{-1} using $\lvert A \rvert$ and *adj* A as: $A^{-1} = \frac{1}{\lvert A \rvert} \times \text{adj } A$ $= \frac{1}{-4} \begin{bmatrix} 1 & -1 \\ -5 & 1 \end{bmatrix}$	**1**

Q.No	What to look for	Marks
	Writes that X = A $^{-1}$ B and finds X as: $\begin{bmatrix} -1 \\ 8 \end{bmatrix}$ **Concludes that *p* = -1 and *q* = 8.**	1
	Writes *h* (*t*) = -*t* ^{2} +8 *t* +30.	0.5
9	**Finds B $^{-1}$ as (AB) $^{-1}$ A as follows:** $\frac{1}{48}\begin{bmatrix} 0 & 0 & 16 \\ 0 & 24 & 0 \\ 3 & 0 & 0 \end{bmatrix}\begin{bmatrix} -2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & \frac{-1}{2} & 0 \\ \frac{-1}{8} & 0 & 0 \end{bmatrix}$	1.5
	Finds adj (B $^{-1}$) as follows: $adj(\mathrm{B}^{-1}) = \begin{bmatrix} 0 & 0 & \frac{-1}{16} \\ 0 & \frac{1}{8} & 0 \\ \frac{1}{2} & 0 & 0 \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{8} & 0 \\ \frac{-1}{16} & 0 & 0 \end{bmatrix}$	1.5
	Finds det (B $^{-1}$) as 0 - 0 + 1($\frac{-1}{16}$) = $\frac{-1}{16}$.	1
	Finds the matrix B as follows: $\mathrm{B} = \frac{1}{\frac{-1}{16}}\begin{bmatrix} 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{8} & 0 \\ \frac{-1}{16} & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -8 \\ 0 & -2 & 0 \\ 1 & 0 & 0 \end{bmatrix}$	1

Chapter - 5
Continuity and Differentiability

Multiple Choice Questions

Q: 1 **The function *f* : R -> R is defined by:**

$$f(x) = \begin{cases} |\sin x|, \text{ when } x \text{ is rational} \\ -|\sin x|, \text{ when } x \text{ is irrational} \end{cases}$$

Read the statements carefully and then choose the option that correctly describes them.

Statement 1 **: *f* (*x*) is continuous at *x* = 0.**

Statement 2 **: *f* (*x*) is discontinuous for all *x* ∈ R - {0}.**

1. Statement 1 is true but Statement 2 is false.
2. Statement 1 is false but Statement 2 is true.
3. Both Statement 1 and Statement 2 are true.
4. Both Statement 1 and Statement 2 are false.

Q: 2 **The Signum function *f* : R -> R, defined below, is discontinuous at *x* = 0. The identity function *g* : R -> R, defined below, is continuous everywhere.**

$$f(x) = \begin{cases} 1, \text{ for } x > 0 \\ 0, \text{ for } x = 0 \\ -1, \text{ for } x < 0 \end{cases}$$

$$g(x) = x, \text{ for all } x \in R$$

Which of these is true about the product of two functions (*f*.*g*)?

1. ($f.g$) is discontinuous at x = 0 but continuous elsewhere in R.
2. ($f.g$) is not differentiable anywhere in R.
3. ($f.g$) is differentiable everywhere in R.
4. ($f.g$) is continuous everywhere in R.

Q: 3 **Read the statements carefully and choose the option that correctly describes them.**

Statement 1: The function $\sqrt{x-5}$ is continuous in $(5, \infty)$.

Statement 1: The function $\sqrt{x-5}$ is differentiablein $(5, \infty)$.

1. Both statements are true and statement 2 explains why statement 1 is true.
2. Both statements are true but statement 2 does NOT explain why statement 1 is true.
3. Statement 1 is true but statement 2 is false.
4. Statement 1 is false but statement 2 is true.

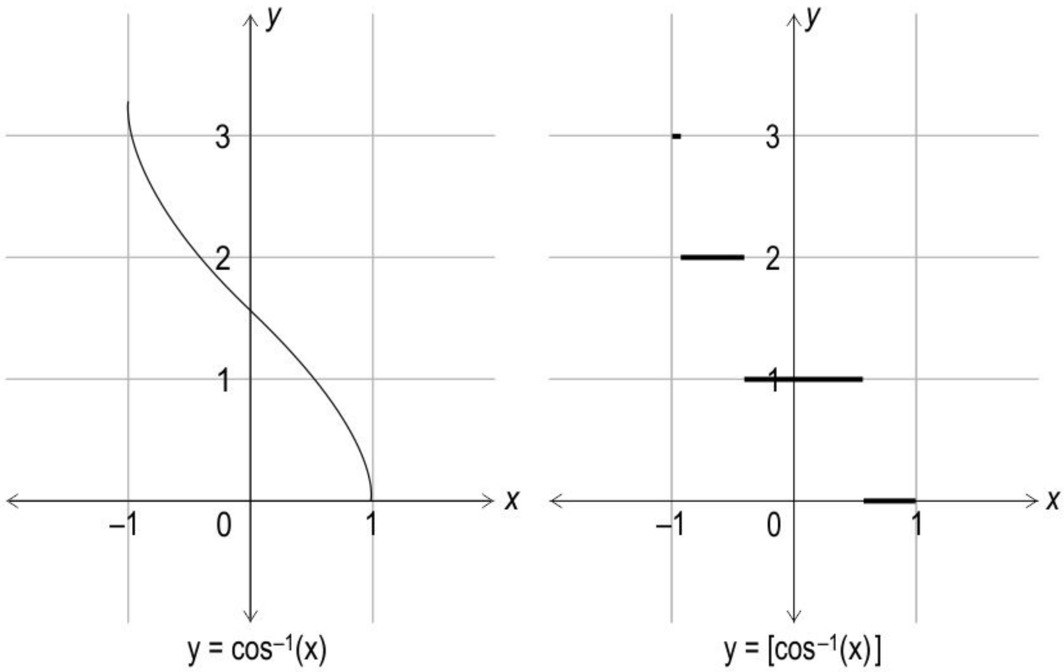
Free Response Questions

Q: 4 **Look at an inverse function below.** **[1]**

$y = \text{cosec}^{-1}(4x^4)$; $|4x^4| > 1$

Find $\frac{dy}{dx}$. Show your steps.

Q: 5 **Shown below are the graphs of two functions $y = \cos^{-1} x$ and $y = [\cos^{-1} x]$, where $[\cos^{-1} x]$ denotes the greatest integer function.** **[2]**



Find the points of discontinuity of the function $y = [\cos^{-1} x]$. Show your work.

Q: 6 If $y = ax^{n+2} + \frac{b}{x^{n+1}}$, where $n \in \mathrm{N}$ and $a, b \in \mathrm{R}$, **[3]**

prove that $x^2 y'' = (n+1)(n+2)y$.

Q: 7 **Find the value of $(f \circ g)'$ at $x = 4$ if $f(u) = u^3 + 1$ and $u = g(x) = \sqrt{x}$. Show your work.** **[3]**

Q: 8 **[5]**

$$f(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^{10}}{10!}$$

If $f(x)$ is differentiated successively 10 times, what is $f^{(10)}(x)$? Show your work.

Q: 9 **[5]**

Find $\frac{d^2y}{dx^2}$ if, $y = tan^{-1}\left[\frac{log\left(\frac{e}{x^5}\right)}{log\left(ex^5\right)}\right] + tan^{-1}\left[\frac{4 + 5\ log\ x}{1 - 20\ log\ x}\right]$.

Show your work.

(*Note : Consider the base of logarithm to be e.*)

Q: 10 **Find $\frac{dm}{dn}$ at $(m, n) = (-1, 1)$ where:** **[5]**

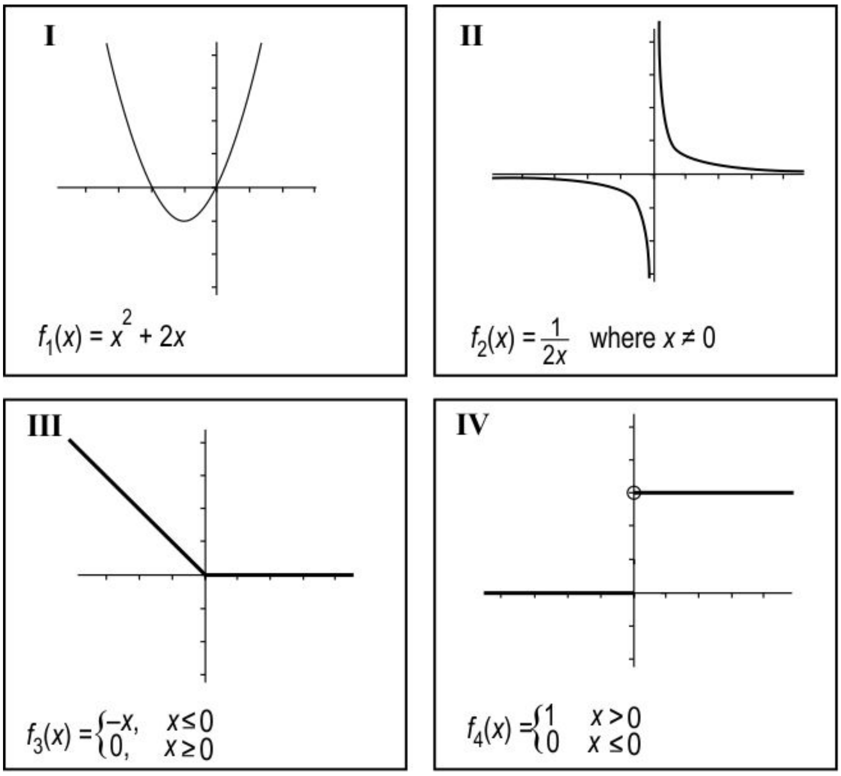
$$5m^2 + 2m - n^{\frac{-2}{3}} = 0$$

Show your work.

Case Study

Answer the questions based on the information given below.

Ambika, a mathematics teacher is conducting a practice session on calculus where she is discussing continuity and differentiability of several functions in their domains. Shown below are four cards that contain a function each along with their domains and graphs.



While discussing in the group, four students claimed as follows:

♦ Leela: "As the function on card I is both continuous and differentiable, we can say that every continuous function is differentiable."
♦ Irfan: "As the graph is not in one piece, the function on card II is discontinuous."
♦ Deepak: "The function on card III is continuous."
♦ Kiran: "The function on card IV is discontinuous."

Q: 11 **Check whether Deepak and Kiran's claims are correct. Justify your answer.** **[2]**

Q: 12 **Is Irfan's claim correct? Does the reason given by him support his claim? Justify.** **[1]**

Q: 13 **Is Leela's claim true for all continuous functions? Justify with a valid reason or provide a counterexample.** **[2]**

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	4
2	4
3	1

Q.No	What to look for	Marks
4	**Differentiates the given function as:** $$\frac{dy}{dx} = \frac{-1}{4x^4\sqrt{(4x^4)^2-1}}\,\frac{d(4x^4)}{dx}$$	0.5
	Simplifies the above equation as: $$\frac{dy}{dx} = \frac{-16x^3}{4x^4\sqrt{(4x^4)^2-1}} = \frac{-4}{x\sqrt{16x^8-1}}$$	0.5
5	**With the help of the graphs, finds the values of x for which $\cos^{-1} x$ attains integer values as:** **$\cos^{-1} x = 3$** **=> $x = \cos 3$** **$\cos^{-1} x = 2$** **=> $x = \cos 2$** **$\cos^{-1} x = 1$** **=> $x = \cos 1$**	1.5
	Concludes that the points of discontinuity of the function $y = [\cos^{-1} x]$ are cos 3, cos 2 and cos 1.	0.5
6	**Finds the first derivative of y as:** $$y' = a(n+2)x^{n+1} - b(n+1)x^{-n-2}$$	1
	Finds the second derivative of y as: $$y'' = a(n+2)(n+1)x^{n} + b(n+1)(n+2)x^{-n-3}$$	1

Q.No	What to look for	Marks
	Multiplies x^2 on both sides of the above equation to get: $x^2y'' = (n+1)(n+2)\left[ax^{n+2} + \frac{b}{x^{n+1}}\right]$ $\Rightarrow x^2y'' = (n+1)(n+2)y$	1
7	**Finds (*f* o *g*)(*x*) as:** $(f \circ g)(x)$ $= f(g(x))$ $= f(\sqrt{x})$ $= (\sqrt{x})^3 + 1$ $= x^{\frac{3}{2}} + 1$	1
	Finds the derivative of (*f* o *g*)(*x*) as: $\frac{d}{dx}(f \circ g)(x)$ $= \frac{d}{dx}(x^{\frac{3}{2}} + 1)$ $= \frac{3}{2}x^{\frac{1}{2}}$	1
	Uses the above step to find the value of (*f* o *g*)' at *x* = 4 as 3.	1

Q.No	What to look for	Marks
8	**Differentiates *f* (*x*) once to get:** $f^{(1)}(x) = 0 + 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdot\cdot\cdot + \frac{x^9}{9!}$	1
	Rewrites *f* $^{(1)}$ (*x*) as: $f^{(1)}(x) = f(x) - \frac{x^{10}}{10!}$	0.5
	Finds the second derivative of *f* (*x*) as: $f^{(2)}(x) = f(x) - \frac{x^{10}}{10!} - \frac{x^9}{9!}$	1
	Finds the third derivative of *f* (*x*) as: $f^{(3)}(x) = f(x) - \frac{x^{10}}{10!} - \frac{x^9}{9!} - \frac{x^8}{8!}$	1
	Generalises the pattern as: $f^{(10)}(x) = f(x) - \frac{x^{10}}{10!} - \frac{x^9}{9!} - \cdots - \frac{x^2}{2!} - x$ **(Award full marks if the expression is correctly generalised at the second derivative stage.)**	1

Q.No	What to look for	Marks
	Finds the value of $f^{(10)}(x)$ as $f^{(10)} = 1$.	0.5
9	Simplifies $\log\left(\frac{e}{x^5}\right)$ as $1 - 5\log x$.	0.5
	Simplifies $\log(ex^5)$ as $1 + 5\log x$.	0.5
	Rewrites the given equation as: $y = \tan^{-1}\left[\frac{1-5\log x}{1+5\log x}\right] + \tan^{-1}\left[\frac{4+5\log x}{1-20\log x}\right]$	0.5
	Uses the trigonometric identities $\tan^{-1}A - \tan^{-1}B = \tan^{-1}\frac{A-B}{A+B}$ and $\tan^{-1}A + \tan^{-1}B = \tan^{-1}\frac{A+B}{1-AB}$ and rewrites the above equation as: $y = \tan^{-1}1 - \tan^{-1}(5\log x) + \tan^{-1}4 + \tan^{-1}(5\log x)$	2
	Simplifies above step to get: $y = \tan^{-1}1 + \tan^{-1}4$	0.5
	Differentiates the above equation to get $y' = 0$.	0.5
	Differentiates y' to get $y'' = 0$.	0.5
10	**Differentiates the equation given in the question with respect to n as follows:** $\frac{d}{dn}\left(5m^2 + 2m - n^{\frac{-2}{3}}\right) = \frac{d}{dn}(0)$ $\Rightarrow \frac{d}{dn}(5m^2) + \frac{d}{dn}(2m) - \frac{d}{dn}\left(n^{\frac{-2}{3}}\right) = 0$	1
	Simplifies the differentiation in the previous step as follows: $\frac{d}{dm}(5m^2)\frac{dm}{dn} + 2\frac{dm}{dn} + \frac{2}{3}n^{\frac{-5}{3}} = 0$	1

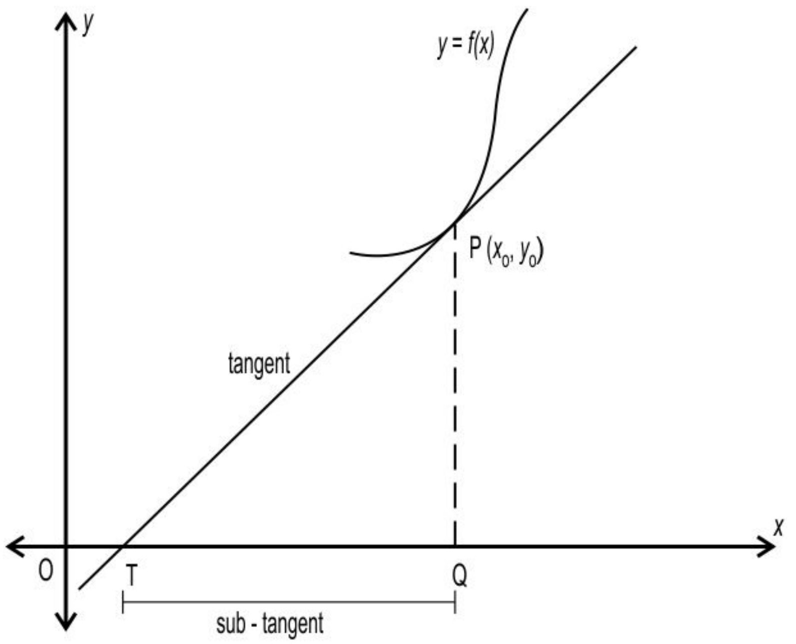
Q.No	What to look for	Marks
	Simplifies the previous step as: $\frac{dm}{dn}(10m+2)+\frac{2}{3}n^{\frac{-5}{3}}=0$ **(Award full marks if equivalent solving techniques without some intermediate steps are followed.)**	**1**
	Concludes that: $\frac{dm}{dn}=-\frac{n^{\frac{-5}{3}}}{15m+3}$	**1**
	Finds the value of $\frac{dm}{dn}$ at (-1,1) as $\frac{1}{12}$.	**1**
11	**Writes that Deepak is right and justifies it as follows:** **As f_3 (x) satisfies all the 3 conditions given below, it is continuous.** ♦ **$f_3(x)$ is defined for all real numbers.** ♦ **At $x = 0$, left hand limit = right hand limit = 0** ♦ **$f(0) = 0$**	**1.5**
	Writes that Kiran is right and justifies it as follows: **As $f_4(x)$ fails to satisfy that at $x = 0$, left hand limit (0) is not equal to the right hand limit (1), it is discontinuous.**	**0.5**
12	**Writes that the claim made by Irfan is wrong and the reason given by him does not support his claim.**	**0.5**
	Justifies that when $x = 0$, the function is not defined. So, $\frac{1}{2x}$ is continuous at all points except when $x = 0$. Hence, writes that the reason given by Irfan is not correct.	**0.5**
13	**Writes that Leela's claim is not true for all continuous functions.**	**0.5**

Q.No	What to look for	Marks
	Gives an example. Foe example: $f_3(x)$ is continuous, but not differentiable as it it has a corner point. (Award full marks if any other valid examples are given.)	1.5

Chapter - 6
Application of Derivatives and multiconcepts

Multiple Choice Questions

Q: 1 **The sub-tangent of a curve at a point is the projection on the *x* -axis of the portion of the tangent to the curve between the *x* -axis and the point of tangency. The sub-tangent of a curve $y = f(x)$ at a point P(x_o, y_o) is illustrated below.**

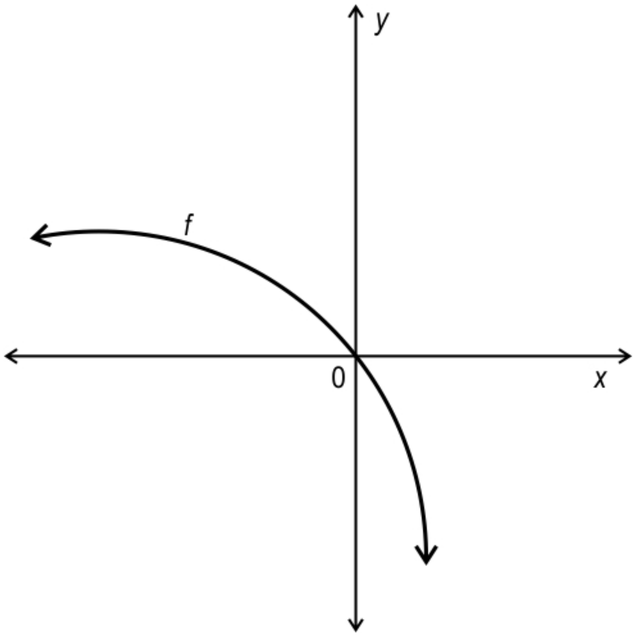


Among the given slopes of tangents of a curve at a given point, which will result in the longest sub-tangent?

1 30° 2 45° 3 60° 4 90°

Free Response Questions

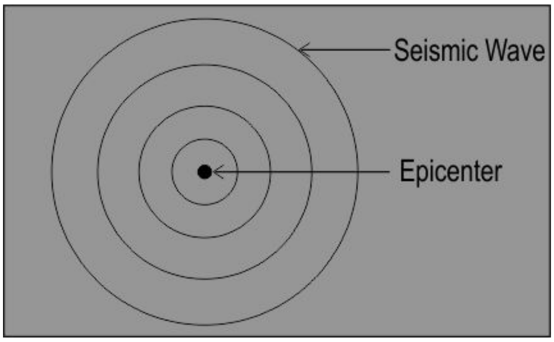
Q: 2 **The graph of a function, *f* : R -> R, is shown below.** **[1]**



Nadeem said that *f* '(0) > *f* (0).

Is Nadeem right? Give a valid reason.

Q: 3 **During an earthquake, seismic waves radiate from the epicenter of an earthquake in a circular pattern, as shown in the figure below.** **[1]**



If seismic waves travel at a speed of approximately 6 km/sec, then what is the rate of change of the area affected by the earthquake when the radius of the affected area is 25 km? Show your steps.

(Note: Take π as 3.14.)

Q: 4 **f(x) is an increasing function on the interval [0, 4], and f'(5) > 0.** **[1]**

Based on this information, is f(x) an increasing function on the interval [0, 5]? Justify your answer.

Q: 5 **A drone is an unmanned remotely operated aerial vehicle, often used for target practice or surveillance.** **[2]**

One such drone is flying according to the equation $s = t^n + 10$, where s is the distance of the drone from its remote's location at time t and n is a real number. The position of the remote is fixed.

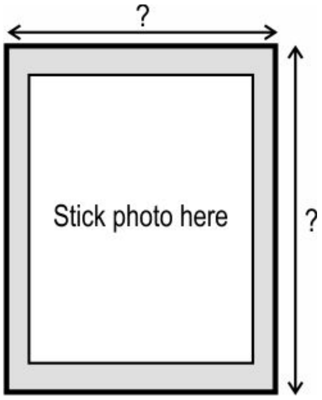
If the velocity of the drone is equal to its acceleration at 3 seconds, find n. Show your work.

Q: 6 **Find the equation of the tangent to the curve $x^2 y^2 = 4$ at the point (1, -2). Show your steps.** **[2]**

Q: 7 **When the diameter of a circle is 8 cm, by what factor does a small change in diameter affect its area? Show your work.** **[2]**

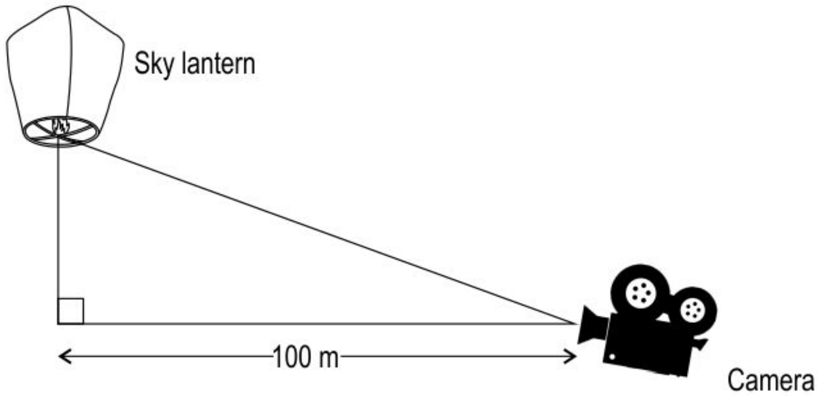
Q: 8 **Mr Aithal, a mathematics teacher, announces the following activity in his classroom and assures grand prizes for the winners.** **[5]**

Instructions:
♦ **Make a rectangular photo frame of total area 80 cm^2 using a chart paper.**
♦ **The frame should have a margin of 1.25 cm each at the top and the bottom.**
♦ **The frame should have a margin of 1 cm each on the left and the right sides.**
♦ **The area available at the centre to stick the photo should be maximum.**



What must be the dimensions of such a photo frame? Show your work.

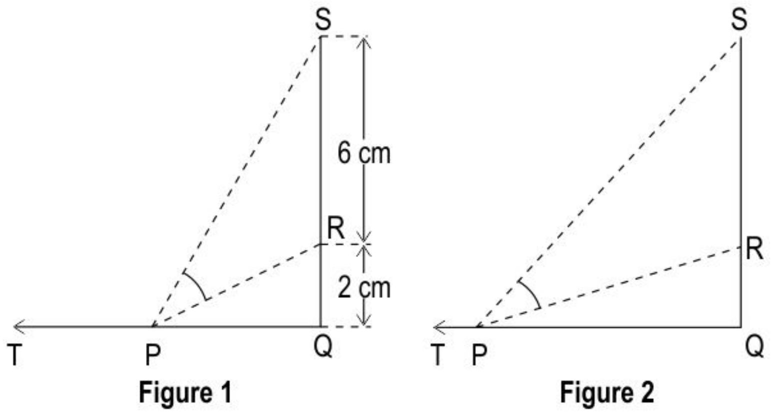
Q: 9 **A camera positioned on the ground recorded a sky lantern that was located 100 meters away. The lantern raised vertically from the ground into the sky at a constant rate of 25 meters per minute, and this entire process was captured on camera.** **[5]**



(Note: The figure is not to scale.)

When the lantern was at 75 m from the ground, what was the rate of change of angle of elevation, in radians/min? Show your steps.

Q: 10 **In the figures shown below, points Q, R and S are fixed. Point P can move forward and backward along ray QT.** **[5]**



(Note: The figures are not to scale.)

What should be the length of PQ such that ∠RPS is maximum? Show your work.

Case Study

Answer the questions based on the given information.

The total cost $C(n)$ of manufacturing n earphone sets per day in the House of Spark Electronics Limited is given by:

$C(n) = 400 + 4n + 0.0001n^2$ dollars.

Each earphone set is sold at:

$q = 10 - 0.0004\,n$ dollars where $n \geq 0$, $q \geq 0$

The daily profit in dollars is determined by the equation:

$P(n) = qn - C(n)$

Q: 11 **The marginal cost, $M(n)$ is the change in total production cost that comes from making or producing one additional unit. It is determined by the instantaneous rate of change of the total cost.** **[2]**

Find the marginal cost $M(n)$ of 10 earphone sets. Show your work.

Q: 12 **What quantity of daily production maximizes the profit? Show your work.** **[3]**

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	1

Q.No	What to look for	Marks
2	**Writes that Nadeem is wrong.**	**0.5**
	Gives a reason. For example, f (0) = 0, and since f is decreasing, f '(0) < 0. Hence, f '(0) < f (0).	**0.5**
3	**Finds the rate of change of the affected area as $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ km^2/sec.**	**0.5**
	Finds the rate of change of the area affected by the earthquake when the radius of the affected area (r) is 25 km as 2 × 3.14 × 25 × 6 = 942 km^2/sec.	**0.5**
4	**Writes that f(x) need not be an increasing function on [0, 5].**	**0.5**
	Gives a reason. For example, even though f'(5) > 0, there may be an x in (4, 5), such that f'(x) < 0.	**0.5**
5	**Differentiates s with respect to time to find the velocity of the drone as:** $s' = nt^{(n-1)}$	**0.5**
	Differentiates s' with respect to time to find the acceleration of the drone as: $s'' = n\ (n - 1)\ t^{(n-2)}$	**0.5**
	Equates velocity of the drone to its acceleration at 3 seconds to find n as: $n \times 3^{(n-1)} = n\ (n - 1) \times 3^{(n-2)}$ $\Rightarrow n \times 3^{(n-1)} = n\ (n - 1) \times 3^{(n-1)} \times 3^{-1}$ $\Rightarrow n = 4$	**1**
6	**Differentiates the given equation using the chain rule as follows:** $2\ xy^2 + (\ x^2\ )\ 2\ y\ (\frac{dy}{dx}) = 0$	**0.5**
	Simplifies the above equation as: $\frac{dy}{dx} = \frac{-y}{x}$	**0.5**

Q.No	What to look for	Marks
	Substitutes (1, -2) in the above equation to get the value of slope (m) as 2.	0.5
	Finds the equation of tangent as: $y - (-2) = 2(x - 1)$ $\Rightarrow y = 2x - 4$	0.5
7	Writes the area of a circle in terms of diameter, D as: $A = \frac{\pi}{4} D^2$	0.5
	Finds the rate of change of area with respect to diameter as follows: $\frac{dA}{dD} = \frac{\pi}{2} D$	1
	Finds $\frac{dA}{dD}$ when D = 8 cm as 4π cm^2/cm. Concludes that for a small change in diameter, the area changes by a factor of 4π. (Award full marks if the problem is solved correctly using approximation concept to obtain $4\pi x$ as the answer, where x is the small change in diameter.)	0.5
8	Assumes the width of photo frame to be x cm and its length to be $\frac{80}{x}$ cm.	0.5
	Subtracts the specified margins from the width and length to find the area (A) available to stick the photo as: $A = (x - 2)(\frac{80}{x} - \frac{5}{2})$ $\Rightarrow A = 85 - \frac{5}{2}x - \frac{160}{x}$	1
	Differentiates area with respect to x as: $\frac{dA}{dx} = -\frac{5}{2} + \frac{160}{x^2}$	1

Q.No	What to look for	Marks
	Equates the above derivative to zero and finds the critical point as: $-\frac{5}{2} + \frac{160}{x^2} = 0$ $\Rightarrow x^2 = 64$ $\Rightarrow x = 8$ (as x being a length cannot be negative)	1
	Finds $\frac{d^2A}{dx^2}$ at $x = 8$ as: $\frac{d^2A}{dx^2}_{(\text{at } x=8)} = -\frac{320}{x^3}_{(\text{at } x=8)} = -\frac{5}{8} < 0$ **Concludes that by second derivative test, the area is maximum at *x* = 8 cm.**	1
	Finds the length of the frame as $\frac{80}{8}$ = 10 cm. **Concludes that the required dimensions of the photo frame are 8 cm and 10 cm.**	0.5
9	**Takes *x* as the distance between the lantern and the ground, θ as camera's angle of elevation in radians and *t* as the time in minutes.** **Writes that $\frac{dx}{dt}$ = 25 m/min.**	0.5
	Uses tangent function and writes: $\tan\theta = \frac{x}{100}$	0.5
	Differentiates the above equation with respect to *t* to get: $\sec^2\theta \times \frac{d\theta}{dt} = \frac{1}{100} \times \frac{dx}{dt}$	1
	Uses steps 1 and 3 to write: $\frac{d\theta}{dt} = \frac{1}{4\sec^2\theta}$	0.5

Q.No	What to look for	Marks
	Uses secant function and writes: **sec θ =** $\frac{y}{100}$ **where *y* is the distance between the camera and the lantern.**	0.5
	Uses the Pythagoras theorem to find *y* as: $y^2 = x^2 + 100^2$ $\Rightarrow y^2 = 75^2 + 100^2$ $\Rightarrow y = 125$, as $y > 0$.	1
	Substitutes *y* = 125 to get sec θ as $\frac{5}{4}$ **.**	0.5
	Substitutes the value of sec θ in the equation obtained in step 3 to get: $\frac{d\theta}{dt} = \frac{4}{25}$ **or 0.16 radians/min.**	0.5
10	**Considers the length of PQ as *x* cm and finds ∠QPR and ∠QPS as:** $\angle QPR = \tan^{-1}\left(\frac{2}{x}\right)$ $\angle QPS = \tan^{-1}\left(\frac{8}{x}\right)$	0.5
	Considers ∠RPS as θ finds θ in terms of *x* as: $\theta = \tan^{-1}\left(\frac{8}{x}\right) - \tan^{-1}\left(\frac{2}{x}\right)$	0.5
	Finds the derivative of θ with respect to *x* as follows: $\frac{d\theta}{dx} = \frac{d}{dx}\left(\tan^{-1}\left(\frac{8}{x}\right) - \tan^{-1}\left(\frac{2}{x}\right)\right)$ $= \frac{d}{dx}\left(\tan^{-1}\left(\frac{8}{x}\right)\right) - \frac{d}{dx}\left(\tan^{-1}\left(\frac{2}{x}\right)\right)$	0.5

Q.No	What to look for	Marks
	Applies chain rule and differentiates as follows: $\frac{1}{\left(\frac{8}{x}\right)^2+1} \cdot \frac{d}{dx}\left(\frac{8}{x}\right) - \frac{1}{\left(\frac{2}{x}\right)^2+1} \cdot \frac{d}{dx}\left(\frac{2}{x}\right)$ $= \frac{2}{\left[\left(\frac{4}{x^2}+1\right)x^2\right]} - \frac{8}{\left[\left(\frac{64}{x^2}+1\right)x^2\right]}$	**1.5**
	Equates the derivative to 0 to find that the maximum value of θ will occur when *x* is either 4 or (-4). States that the minimum value of θ is 0, which cannot occur at *x* = 4 and hence it must be the maxima. The working may look as follows: $\frac{8}{\left[\left(\frac{64}{x^2}+1\right)x^2\right]} = \frac{2}{\left[\left(\frac{4}{x^2}+1\right)x^2\right]}$ Cancelling x^2 on both sides, $4\left(\frac{4}{x^2}+1\right) = \frac{64}{x^2}+1$ Rearranges terms to obtain, $x^2 = 16$ $\Rightarrow x = +4 \text{ or } -4$	**1.5**
	Ignores *x* = -4 as the length of PQ cannot be negative and writes ∠RPS will be maximum when length of PQ = 4 cm.	**0.5**
11	**Writes the marginal cost function as:** $M(n) = \frac{dC}{dn} = \frac{d}{dn}(400 + 4n + 0.0001n^2)$	**0.5**
	Finds the marginal cost function by completing the differentiation in the above step as: $M(n) = 4 + 0.0002n$	**1**
	Finds the marginal cost of 10 earphone sets as 4.002 dollars by substituting 10 for *n* in the equation obtained in the above step.	**0.5**
12	**Writes the daily profit function as:** $P(n) = (10 - 0.0004n)n - (400 + 4n + 0.0001n^2)$	**0.5**

Q.No	What to look for	Marks
	Simplifies the above equation to find the daily profit function as: $P(n) = -0.0005n^2 + 6n - 400$	**0.5**
	Differentiates the daily profit function obtained in the previous step as follows: $P'(n) = \frac{d}{dn}(-0.0005n^2 + 6n - 400)$ $= -0.001n + 6$	**0.5**
	Finds the critical point of $P(n)$ as $n = 6000$ by equating $P'(n)$ to 0 and solving for n as shown below: $P'(n) = -0.001n + 6 = 0$ $\Rightarrow n = 6000$	**0.5**
	Finds the second derivative as: $P''(n) = -0.001$ **Writes that this means that the function will be at its maximum at $n = 6000$.**	**0.5**
	Concludes that since the maximum value is obtained at $n = 6000$, hence the profit is maximized when the daily production is 6000 earphones.	**0.5**

Chapter - 7
Integrals

Multiple Choice Questions

Q: 1 **Look at the integral given below.**

$$\int_{\frac{a}{4}}^{\frac{b}{4}} f(4x)dx$$

If $f(x)$ is continuous for all real values of x, then which of these is equal to the above integral?

1. $4\int_{a}^{b} f(x)dx$
2. $\frac{1}{4}\int_{a}^{b} f(x)dx$
3. $\frac{1}{4}\int_{4a}^{4b} f(x)dx$
4. $4\int_{4a}^{4b} f(x)dx$

Q: 2 **What is the value of the following integral?**

$$\int_{-2}^{2} (2-|x|)\,dx$$

1. 0
2. 4
3. 8
4. 12

Q: 3 **Varath says the following:**

$$\int_{-3}^{3} \left(\sqrt{x^2-4}\right) dx = F(3) - F(-3),$$

where $F(x)$ is the antiderivative of $\left(\sqrt{x^2-4}\right)$.

Which of the following can be said about Varath's statement?

1. It is true, as the function is continuous in [-3,3].
2. It is true, as per the fundamental theorem of calculus.
3. It is false, as the integral is not defined over the interval.
4. It is false, as the antiderivative of any function within a square root does not exist in R.

Free Response Questions

Q: 4 **Integrate the following function with respect to x.** **[1]**

$$e^{6-\ln x}$$

Show your steps.

Q: 5 **Given:** **[1]**

$$\int \frac{dx}{f(x)} = \frac{1}{3}\tan^{-1}\left(\frac{x-4}{3}\right) + C$$

where C is an arbitrary constant.

Find *f* (*x*). Show your work.

Q: 6 Check whether the given statement is true or false. **[1]**

For any function $f(x)$ that satisfies the condition $f(-3) = -f(3)$,
$\int_{-3}^{3} f(x) = 0$

Justify your answer.

Q: 7 **Ankit's partial solution for a question on integration is given below.** **[2]**

Question: Solve $\int \cos^3 x \sin^2 x\, dx$.

Ankit's solution:
Step 1: Let I = $\int \cos^3 x \sin^2 x\, dx$
Step 2: I = $\int \cos^3 x(1 - \cos^2 x)\, dx$
Step 3: I = $\int (\cos^3 x - \cos^5 x)\, dx$
Step 4: Let $\cos x = t$
Step 5: I = $\int (t^3 - t^5)\, dt$

Is Ankit correct? If yes, complete the integration. If no, in which step is the error present? Explain your reasoning.

Q: 8 **If *h* '(*x*) = *g* (*x*) and *g* is a continuous function for all real values of *x*, then prove that:** **[2]**

$$\int_{-1}^{1} g(6x)dx = \frac{1}{6}h(6) - \frac{1}{6}h(-6)$$

Q: 9 **Solve:** **[3]**

$$\int \frac{1}{m^2} \cos^2\left(\frac{1}{m} - 1\right) dm$$

Show your steps.

Q: 10 **Evaluate the integral:** **[3]**

$$I = \int_0^{\frac{\sqrt{3}}{2}} \left[\frac{\sin^{-1}x}{(1-x^2)^{\frac{3}{2}}} dx\right]$$

Show your work.

Q: 11 **Solve the following integration and write your answer in its most simplified form.** **[5]**

$$\int \frac{x^2}{e^{2x}} dx$$

Show your steps.

Q: 12 **Integrate the given function. Show your steps.** **[5]**

$$\int \cot^{-1}\left(\frac{5}{x}\right) dx$$

Q: 13 Integrate $\frac{2e^x}{(e^x-1)(2e^x+3)}$. **[5]**

Show your work.

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	2
2	2
3	3

Q.No	What to look for	Marks
4	**Rewrites the given expression as:** $\frac{e^6}{x}$	**0.5**
	Integrates the above expression with respect to *x* as: $e^6 \ln\lvert x\rvert + C$ **where, C is the constant of integration.**	**0.5**
5	**Finds *f* (*x*) as:** $9 + (x - 4)^2$ **by differentiating the RHS of the given equation using the differentiation of** $\tan^{-1} x$ **as follows:** $\frac{d}{dx}\left(\frac{1}{3}\tan^{-1}\left(\frac{x-4}{3}\right)\right) = \frac{1}{9+(x-4)^2}$	**1**
6	**Writes false.**	**0.5**
	Reasons that this would only be true if $f(-x) = -f(x)$ **for every value of *x*.**	**0.5**
7	**Writes that Ankit is incorrect.**	**0.5**
	Writes that Ankit made an error in step 5.	**0.5**
	Gives a reason. For example, on substituting cos *x* as *t*, one also has to substitute *dx* in terms of *dt,* which Ankit has not done. Instead, he has simply replaced *dx* with *dt*.	**1**
8	**Substitutes** $u = 6x$ **and finds** du **as** $6\,dx$**.**	**0.5**
	Finds the limit as: **When** $x = -1$**, then** $u = -6$**.** **When** $x = 1$**, then** $u = 6$**.**	**0.5**

Q.No	What to look for	Marks
	Rewrites the integral and proves the given statement as: $\int_{-1}^{1} g(6x)dx$ $= \frac{1}{6}\int_{-6}^{6} g(u)du$ $= \left[\frac{1}{6}h(u)\right]_{-6}^{6}$ $= \frac{1}{6}h(6) - \frac{1}{6}h(-6)$	1
9	**Substitutes ($\frac{1}{m}$ - 1) as *u* to get:** $du = -\frac{1}{m^2}dm$	0.5
	Rewrites the given integral as: $-\int \cos^2 u\, du$	0.5
	Substitutes: $\cos^2 u = \frac{(1+\cos 2u)}{2}$ **in the above integral and rewrites it as follows:** $-\frac{1}{2}\int (1 + \cos 2u)\, du$	0.5
	Integrates the above integral to get the following expression where C is the arbitrary constant: $-\frac{1}{2}\left(u + \frac{1}{2}\sin 2u\right) + C$	1

Q.No	What to look for	Marks
	Substitutes *u* as ($\frac{1}{m}$ - 1) in the above expression to get the following expression as the solution: $-\frac{1}{2}\left[\frac{1}{m}-1+\frac{1}{2}\sin 2\left(\frac{1}{m}-1\right)\right]+C$	**0.5**
10	**Takes *u* = sin $^{-1}$ *x*** **Finds du as:** $du = \frac{dx}{\sqrt{1-x^2}}$	**0.5**
	Finds the change in limit when *x* = 0 and *x* = $\frac{\sqrt{3}}{2}$ to *u* = 0 and *u* = $\frac{\pi}{3}$ respectively.	**0.5**
	Rewrites the given integral using the above substitution and integrates the same as: $\mathrm{I} = \int_0^{\frac{\pi}{3}} \frac{u}{\cos^2 u} du$ $\Rightarrow \mathrm{I} = \int_0^{\frac{\pi}{3}} u \sec^2 u \, du$ $\Rightarrow \mathrm{I} = u \tan u \,\Big\|_0^{\frac{\pi}{3}} - \int_0^{\frac{\pi}{3}} \tan u \, du$ $\Rightarrow \mathrm{I} = u \tan u \,\Big\|_0^{\frac{\pi}{3}} + \log \cos u \,\Big\|_0^{\frac{\pi}{3}}$	**1.5**
	Applies the limit to find the value of the given definite integral as: $\frac{\pi}{\sqrt{3}} - \log 2$	**0.5**
11	Let $I = \int \frac{x^2}{e^{2x}} dx$ **Evaluates integral using formula for integral by parts to get the following:** $I = x^2\left(\int e^{-2x} dx\right) - \int \left[2x \int e^{-2x} dx\right] dx$	**1**

Q.No	What to look for	Marks
	Solves the integration of *e* $^{-2x}$ by substituting *u* as (-2 *x*). **Gets *dx* as $\frac{-1}{2}$ *du.*** **The integration may look as follows:** $-\frac{1}{2}\int e^u du = -\frac{1}{2}e^u + c$ $\Rightarrow \int e^{-2x}dx = -\frac{1}{2}e^{-2x} + c$ **where c is a constant.**	1.5
	Substitutes the value of the above integral in the equation from step 2 to get the following: $I = -\frac{x^2}{2e^{2x}} + \int xe^{-2x}dx$	0.5
	Applies integration by parts to solve the integration of *xe* $^{-2x}$ in a similar way as in step 3. **The integration may look as follows:** Let $h(x) = x$ and $g(x) = e^{-2x}$ $\Rightarrow \int xe^{-2x}dx = \frac{x}{-2e^{2x}} - \int\left[\int e^{-2x}dx\right]dx$ $\Rightarrow \int xe^{-2x}dx = -\frac{x}{2e^{2x}} + \frac{1}{2}\int e^{-2x}dx$ $\Rightarrow \int xe^{-2x}dx = -\frac{x}{2e^{2x}} - \frac{1}{4e^{2x}} + c_1$ where c_1 is a constant	1.5
	Writes answer as follows: $I = -\frac{2x^2+2x+1}{4e^{2x}} + C$ **where C is a constant.**	0.5

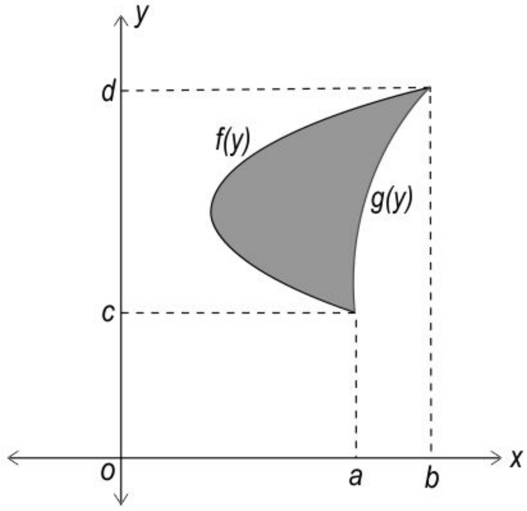
Q.No	What to look for	Marks
12	**Uses integration by parts to rewrite the given integral as:** $\cot^{-1}\left(\frac{5}{x}\right)\int dx - \int\left[\frac{d}{dx}\left(\cot^{-1}\left(\frac{5}{x}\right)\right)\int dx\right]dx$	1
	Simplifies the differentiation of cot $^{-1}$ ($\frac{5}{x}$) in the above expression as: $\frac{d}{dx}\cot^{-1}\left(\frac{5}{x}\right)$ $= -\frac{1}{1+\left(\frac{5}{x}\right)^2}\frac{d}{dx}\left(\frac{5}{x}\right)$ $= \frac{\frac{5}{x^2}}{1+\left(\frac{5}{x}\right)^2}$ $= \frac{5}{x^2+25}$	1.5
	Substitutes the above expression in step 1 and integrates the integral in step 1 to get the following expression: $x\cot^{-1}\left(\frac{5}{x}\right) - 5\int\frac{x}{x^2+25}dx$	1
	Completes integrating the above expression to get: $x\cot^{-1}\left(\frac{5}{x}\right) - \frac{5}{2}\log\left\|x^2+25\right\| + C$ **where C is an arbitrary constant.** **(Award full marks even if modulus is not used in log function as (x^2 + 25) is always positive.)**	1.5
13	Considers $e^x = t$ and finds $dx = \frac{dt}{t}$.	1

Q.No	What to look for	Marks
	Writes the function to be integrated as $\int \frac{2}{(t-1)(2t+3)} dt$.	1
	Uses partial fractions to rewrite the function as follows: $\frac{2}{5} \int \left(\frac{1}{t-1} - \frac{2}{2t+3} \right) dt$	1.5
	Integrates the function to obtain the following: $\frac{2}{5} [\ln\|t-1\| - \ln\|2t+3\|] + C$ **where C is the constant of integration.**	1
	Replaces t with e^x to obtain $\frac{2}{5} \ln\left\|\frac{e^x - 1}{2e^x + 3}\right\| + C$.	0.5

Chapter - 8
Application of Integrals

Multiple Choice Questions

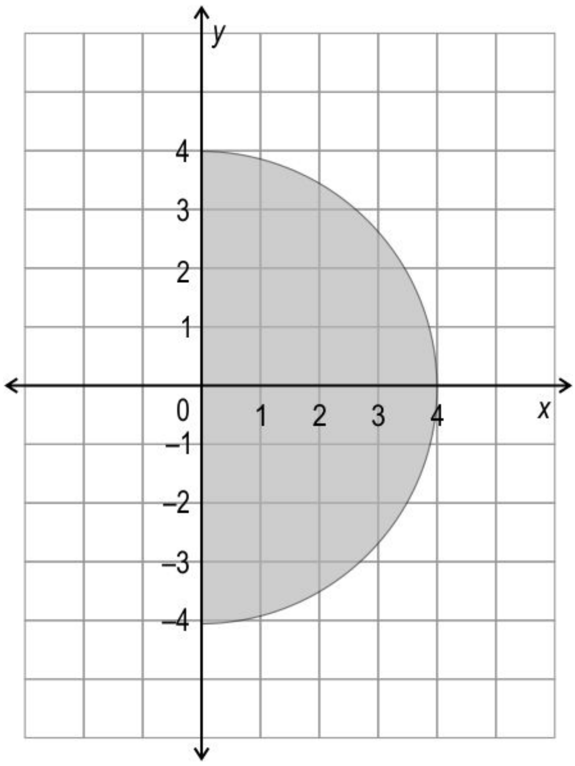
Q: 1 **Shown below are partial graphs of two distinct functions, *f* (*y*) and *g* (*y*). The region between them is shaded.**



Which expression gives the area of this shaded region?

1. $\int_a^b |f(y) - g(y)|\, dy$
2. $\int_a^b |f(x) - g(x)|\, dx$
3. $\int_c^d |f(y) - g(y)|\, dy$
4. $\int_c^d |f(x) + g(x)|\, dx$

Q: 2 **Shown below is the graph of the function $x^2 + y^2 = 16$, $0 \le x \le 4$.**

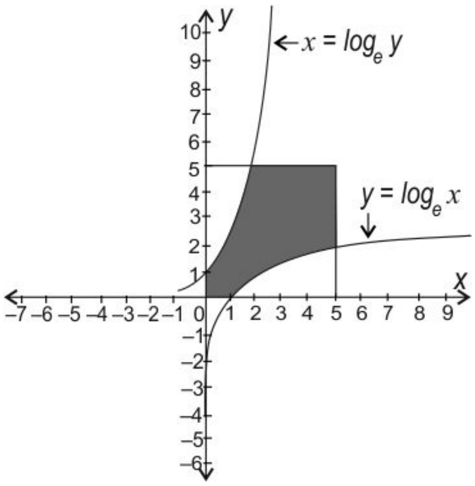


Which of the following expressions represents the area of the shaded region?

1. $2\int_0^4 \sqrt{16 - y^2}dy$
2. $\left|\int_{-4}^0 (16 - y^2)\, dy\right| + \int_0^4 (16 - y^2)\, dy$
3. $\left|\int_{-4}^0 \sqrt{16 - x^2}\, dx\right| + \int_0^4 \sqrt{16 - x^2}\, dx$
4. $\int_0^4 \sqrt{16 - x^2}\, dx$

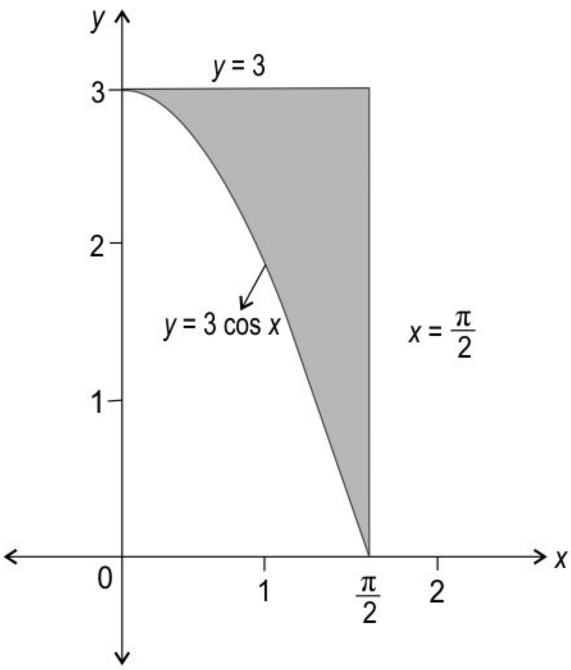
Free Response Questions

Q: 3 **Look at the graph below.** **[1]**



Write an expression for the area of the shaded region.

Q: 4 **Find the area of the shaded region in the graph shown below.** **[1]**



Show your work.

Q: 5 **The area bounded by the lines 2 y = x - 2, x = 1 and x = 4 can be found as follows:** **[1]**

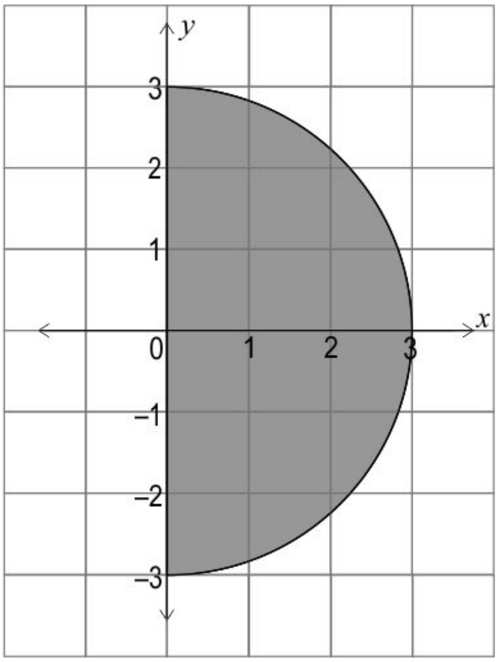
$$\int_1^4 \left(\frac{1}{2}x - 1\right) dx$$

State true or false. If true, justify your answer. If false, write the correct expression for the area.

Q: 6 **Roshan wants to calculate the area bounded by the curve $x^2 + |y| = 4$.** **[1]**

Write an expression that can be used to calculate the area correctly. Justify your answer.

Q: 7 **Shown below is the graph of the function $x^2 + y^2 = 9$, $0 \le x \le 3$.** **[1]**



Rajiv, Swara and Zaman represented the area of the shaded region in the following ways:

Rajiv: $\int_{-3}^{3} \sqrt{9 - x^2}dx$

Swara: $2\int_{0}^{3} \sqrt{9 - y^2}dy$

Zain: $\left|\int_{-3}^{0} \sqrt{9 - x^2}dx\right| + \int_{0}^{3} \sqrt{9 - x^2}dx$

Who represented it correctly and who did not? Justify your answer with a valid reason in each case.

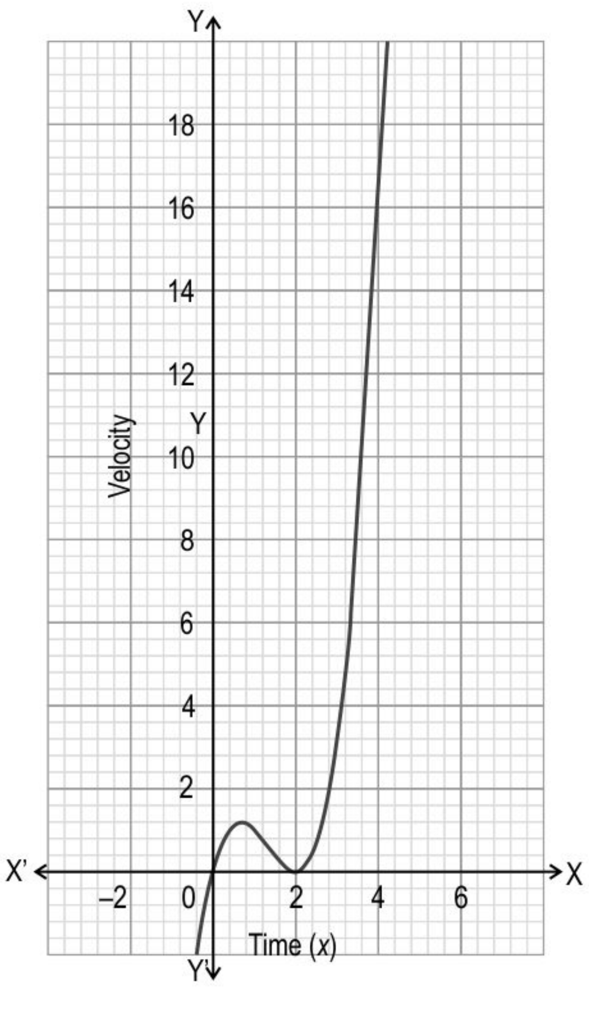
Q: 8 **The velocity function of an object is $v(t) = t^2 + 4t - 5$, where t is in hours and $v(t)$ is in kilometers per hour.** **[2]**

Find the displacement of the object during the first 3 hours. Show your steps.

Q: 9 **The area of the region bounded by $y = 4x^3 - kx^2 + 1$ and the x-axis, between the lines $x = 0$ and $x = 2$ is 10 sq units, where $k \in \mathbb{R}$.** **[2]**

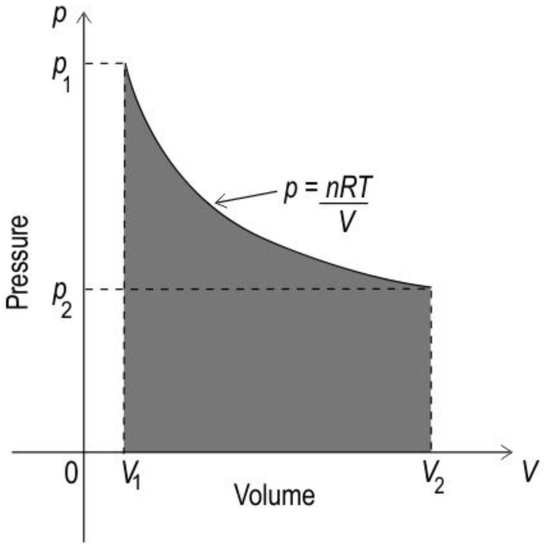
Find the value of k. Show your steps.

Q: 10 **The velocity of a particle, $v(x)$ is given by $(x - 2)^3 + 2(x - 2)^2$ m/s, where x is the time, as shown in the graph below.** **[2]**



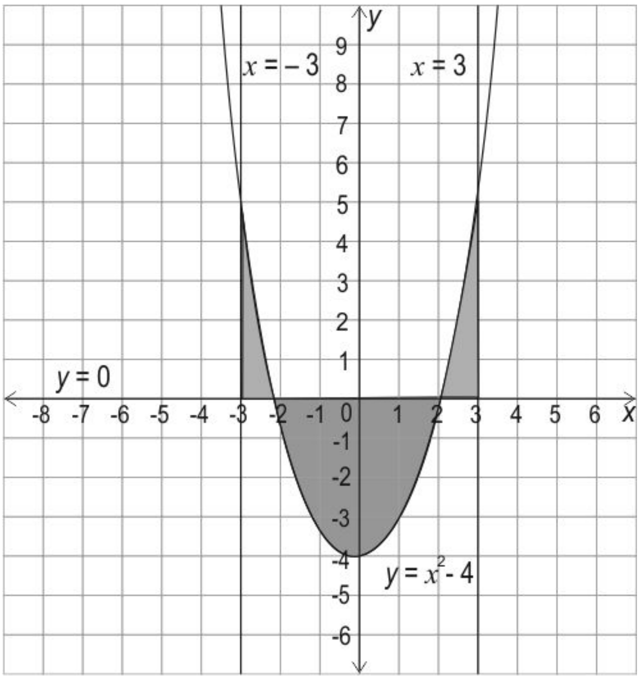
Find the displacement of the particle from 0 to 4 seconds. Show your steps.

Q: 11 **As n moles of a gas undergoes an expansion under constant temperature T, its volume V increases and it does some work on its surroundings. This work is represented by the area under the curve shown in the pressure-volume (pV) diagram below:** **[2]**



Find the expression for the amount of work done by the gas if $pV = nRT$, where R is the gas constant. Show your work.

Q: 12 **The shaded region shown in the graph below represents the area bounded by the graph of $f(x) = x^2 - 4$, the lines $x = -3$, $x = 3$ and the x-axis.** **[5]**



Find the area of the shaded region. Show your steps.

Q: 13 **A certain region is bounded by the x-axis and the graph of $y = \sin\frac{x}{2}$ between $x = 0$ and $x = 2\pi$. It is then divided into 2 regions by the line $x = k$.** **[5]**

If the area of the region between $x = 0$ and $x = k$ is thrice the area of the region between $x = k$ and $x = 2\pi$, find k. Draw a rough figure and show your steps.

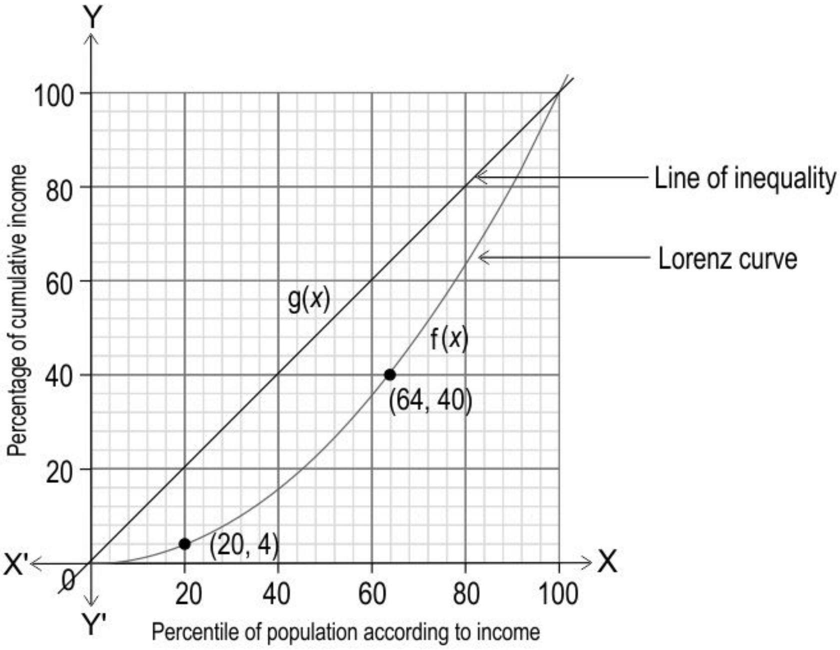
Case Study

Answer the following questions based on the given information.

A Lorenz curve is used to graphically represent income inequality in a society. It was developed by Max Lorenz in 1905.

In this curve, the percentile of the population according to their income is plotted on the x axis, and on the y axis, the percentage of cumulative income from that percentile of the population is plotted.For example, in the graph below, the point (20, 4) denotes that people with more income than that of 20% of the total population contribute 4% of the total income for the country. Similarly, the point (64, 40) denotes that people with more income than that of 64% of the country's population contribute 40% of the total income for the country.
On the graph, there is also a line of equality, given by the function $g(x) = x$. The further away the Lorenz curve of a society is from the line of equality, the more unequal its income distribution is.

In order to compare this data for multiple countries, the area under the Lorenz Curve is used to find the Gini coefficient (G), whose value ranges between 0 and 1. The closer G's value is to 1, the more unequal the income distribution of the society is.



The Lorenz curve shown above can be approximated by the following function:

$$f(x) = \frac{\sqrt{x}}{100} + \frac{(x-1)^2}{100}$$

Q: 14 **Find the area under the Lorenz Curve from 0 to 100. Show your work.** **[3]**

Q: 15 **The Gini coefficient is given by G = $\frac{A}{A+B}$, where A is the area between the Lorenz curve and the line of equality, and B is the area under the Lorenz curve.** **[2]**

Find the Gini coefficient for the given Lorenz curve, using integration. Show your work.

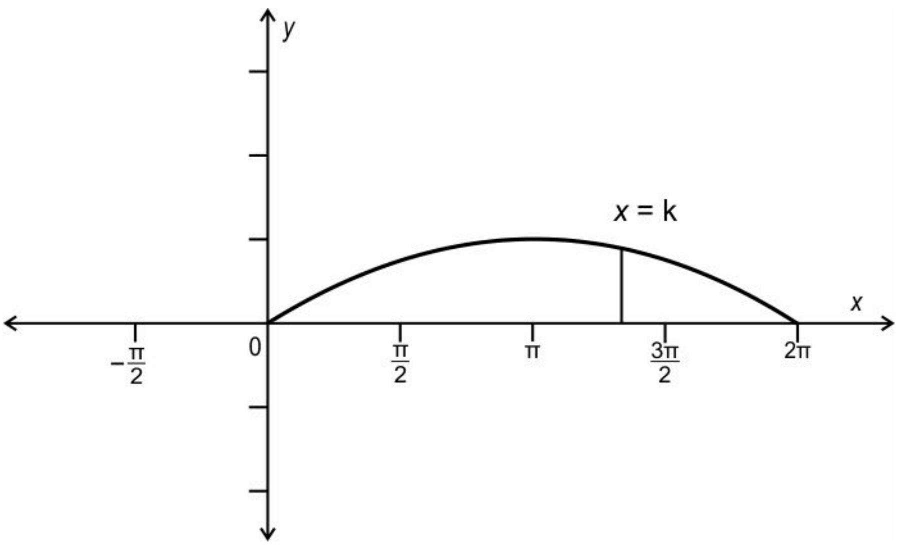
The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	3
2	1

Q.No	What to look for	Marks
3	**Writes an expression for the area of the shaded region as:** $\text{Shaded area} = 25 - 2\int_1^5 \log_e x\,dx$ **(Award full marks for any equivalent expression.)**	1
4	**Represents the area of the shaded region as:** $\frac{3\pi}{2} - \int_0^{\frac{\pi}{2}} 3\cos x dx$	0.5
	Simplifies the above expression to find the area of the shaded region as follows: $\frac{3\pi}{2} - 3\left[\sin x\right]_0^{\frac{\pi}{2}}$ $= 3\left[\frac{\pi}{2} - 1\right]$	0.5
5	**Writes false.**	0.5
	States that the line 2 $y = x$ - 2 crosses the x -axis at $x = 2$ and hence the area under the curve will have to be found as follows: $\left\lvert\int_1^2 (\frac{1}{2}x - 1)\mathrm{d}x\right\rvert + \int_2^4 (\frac{1}{2}x - 1)\mathrm{d}x$	0.5
6	**Writes that the area is equivalent to the area bounded by $x^2 + y = 4$ and $x^2 - y = 4$.**	0.5
	Writes the following expression to calculate the bounded area: $\int_{-2}^{2} (4 - x^2)\ \mathrm{d}x + \left\lvert\int_{-2}^{2} (x^2 - 4)\ \mathrm{d}x\right\rvert$ **(Award full marks for any equivalent expression.)**	0.5

Q.No	What to look for	Marks
7	**Writes that only Swara represented the area of the shaded region correctly. Justifies the answer. For example, the parts of shaded region above and below the *x* -axis are equal. Hence the required area can be computed by integrating the function with respect to *y* -axis from 0 to 3 and then doubling it.**	0.5
	Writes that Rajiv and Zaman represented the area of the shaded region incorrectly. Justifies the answer. For example, if integrated along *x* -axis, the limit must range from 0 to 3. But Rajiv and Zaman have integrated from (-3) to 3.	0.5
8	**Writes the expression for the displacement of the object using the velocity function as follows:** $\int_0^3 (t^2 + 4t - 5)dt$	0.5
	Evaluates the above definite integral as: $\left[\frac{t^3}{3} + \frac{4t^2}{2} - 5t\right]_0^3$	1
	Applies the given limit to find the displacement as 9 + 18 - 15 = 12 km.	0.5
9	**Sets the integral equation as:** $\int_0^2 (4x^3 - kx^2 + 1)dx = 10$	0.5
	Simplifies the above equation as: $\left[\frac{4x^4}{4} - \frac{kx^3}{3} + x\right]_0^2 = 10$	0.5
	Applies the limit and simplifies the above equation as: $16 - \frac{8k}{3} + 2 = 10$	0.5

Q.No	What to look for	Marks
	Simplifies the above equation to get *k* as 3.	0.5
10	Integrates $\int_0^4((x-2)^3+2(x-2)^2)dx$ as $\left[\frac{(x-2)^4}{4}\right]_0^4+\left[\frac{2(x-2)^3}{3}\right]_0^4$	1
	Evaluates the above integral as $\frac{32}{3}$ and hence finds the displacement of the particle from 0 to 4 seconds as $\frac{32}{3}$ m.	1
11	**Represents the work done by the gas as:** $\int_{V_1}^{V_2} pdV$ $= nRT\int_{V_1}^{V_2}\frac{1}{V}dV$	1
	Integrates the above expression to find the work done by the gas as: $nRT\,[\log V]_{V_1}^{V_2}$ $= nRT\,[\log V_2 - \log V_1]$ $= nRT\log\left(\frac{V_2}{V_1}\right)$	1
12	**Sets up the integrals as follows:** Area $= \int_{-3}^{-2}(x^2-4)dx + \left\|\int_{-2}^{2}(x^2-4)dx\right\| + \int_2^3(x^2-4)dx$	1
	Evaluates the area from *x* = -3 to *x* = -2 as follows: $\int_{-3}^{-2}(x^2-4)dx = \left[\frac{x^3}{3}-4x\right]_{-3}^{-2} = \frac{7}{3}$	1

Q.No	What to look for	Marks
	Evaluates the area from *x* = -2 to *x* = 2 as follows: $\left\|\int_{-2}^{2}(x^2-4)dx\right\| = \left\|\left[\frac{x^3}{3}-4x\right]_{-2}^{2}\right\| = \left\|\frac{-32}{3}\right\| = \frac{32}{3}$	**1.5**
	Evaluates the area from *x* = 2 to *x* = 3 as follows: $\int_{2}^{3}(x^2-4)dx = \left[\frac{x^3}{3}-4x\right]_{2}^{3} = \frac{7}{3}$	**1**
	Adds the area found in the above 3 steps to find the area of the shaded region as: $\frac{7}{3}+\frac{32}{3}+\frac{7}{3}=\frac{46}{3}$ or 15.33 sq units	**0.5**
13	**Draws a rough graph of sin $\frac{\pi}{2}$ and *x* = *k*.**  **(Award only 0.5 marks if the range on the *x* -axis is not from 0 to 2π.)**	**1**

Q.No	What to look for	Marks
	Writes the equation for the given situation as: $\int_0^k sin\frac{x}{2}dx = 3\int_k^{2\pi} sin\frac{x}{2}dx$	1
	Simplifies the above equation as: $\left[-2\ cos\frac{x}{2}\right]_0^k = 3\left[-2\ cos\frac{x}{2}\right]_k^{2\pi}$	1
	Simplifies the above equation as: $-2 \cos \frac{k}{2} + 2 \cos 0 = -6 \cos \pi + 6 \cos \frac{k}{2}$	1
	Simplifies the above equation to get *k* as: $\frac{k}{2} = \cos^{-1} \frac{-1}{2} = \frac{2\pi}{3}$ $\Rightarrow k = \frac{4\pi}{3}$	1
14	**Rewrites the integral of f(*x*) as follows:** $\int_0^{100}(\frac{\sqrt{x}}{100} + \frac{(x-1)^2}{100})dx$ $= \int_0^{100} \frac{\sqrt{x}}{100}dx + \int_0^{100} \frac{(x-1)^2}{100}dx$	0.5
	Solves $\int_0^{100} \frac{\sqrt{x}}{100}dx = \left[\frac{2x^{\frac{3}{2}}}{300}\right]_0^{100}$ **Evaluates this to find the answer as $\frac{20}{3}$.** **(Award only 0.5 marks for correctly solving the integration, but incorrectly evaluating the expression.)**	1

Q.No	What to look for	Marks
	Solves $\int_0^{100} \frac{(x^2-2x+1)}{100} dx = \left[\frac{x^3}{300}\right]_0^{100} - \left[\frac{2x^2}{200}\right]_0^{100} + \left[\frac{x}{100}\right]_0^{100}$ **Evaluates this to find the answer as $\frac{10000}{3}$ - 100 + 1 = $\frac{10000}{3}$ - 99.** **(Award only 0.5 marks for correctly solving the integration, but incorrectly evaluating the expression.)**	**1**
	Adds the values from steps 2 and 3 to get: **$\frac{20}{3}$ + $\frac{10000}{3}$ - 99 = 3241 sq units.**	**0.5**
15	**To find A + B, integrates the area under g(x) = x as 5000 sq units.** **The integration may look as follows:** $\int_0^{100} x dx = \left[\frac{x^2}{2}\right]_0^{100} = 5000$	**1**
	Finds A as (5000 - 3241) = 1759 sq. units.	**0.5**
	Calculates G = 1759/5000 = approximately 0.35.	**0.5**

Chapter - 9
Differential Equations

Free Response Questions

Q: 1 **Sumit finds the general solution of the differential equation $yy' = e^{3x}$ as $y = \frac{1}{3}e^{3x} + C$, where C is the arbitrary constant.** **[1]**

Did Sumit find the correct general solution? Show your work and justify.

Q: 2 **The bottom valve of a conical tank is opened to remove sugarcane juice in a factory. The rate at which the juice pours out from the conical tank is directly proportional to the cube root of the rate of change of height of the juice present in the tank.** **[1]**

If k is the constant of proportionality, write a differential equation depicting the scenario.

Q: 3 **A differential equation is given below.** **[1]**

$$\left(\frac{d^3y}{dx^3}\right)^3 + \left(\frac{d^4y}{dx^4}\right)^2 - \left(\frac{dy}{dx}\right)^4 + 7y = 21$$

State whether the following statement is true or false. Justify your answer.

The degree of the given differential equation is equal to its order.

Q: 4 **[1]**

$$\left(\frac{d^4y}{dx^4}\right)^5 + \left(\frac{d^3y}{dx^3}\right)^6 - \left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2 = 25$$

A differential equation is given above. State whether the following statement is true or false. Justify your answer.

The general solution of the given differential equation will have five arbitrary constants.

Q: 5 **If the tangent to a curve at every single point on it is given by $y - \frac{2y}{x+1}$ then find the equation of the curve. Show your steps.** **[2]**

Q: 6 **In a controlled condition within a laboratory, a spherical balloon is being deflated at a rate proportional to its surface area at that instant. The spherical shape of the balloon is maintained throughout the process.** **[2]**

Form a differential equation that represents the rate of change of its radius. Show your work.

Q: 7 **What family of curves does the solution of the differential equation $2x\frac{dy}{dx} = 7 + y$ represent? Show your work.** **[2]**

Q: 8 **The solution of the differential equation $xdy = (3x - 2y)\,dx$ is in the form:** **[3]**

$x^3 = C.\,f(x, y)$, where C is an arbitrary constant.

Find $f(x, y)$. Show your work.

Q: 9 **Birds are sensitive to microwaves that are emitted by mobile phones, which has resulted in a decline in the bird population, especially sparrows.** **[5]**

The population of sparrows in a certain region is decreasing according to the following equation due to the extensive use of mobile phones.

$\frac{dy}{dt} = ky$

where, y represents the population of sparrows at time t (in years) and k is a constant.

The population, which was e^{10} five years ago, has decreased by 25% in that time. Find k. Show your steps.

(Note: Take ln 3 ≈ 1.09 and ln 4 ≈ 1.38.)

Q: 10 **i) Find the differential equation representing the family of curves $(y - c)^2 = x^3$, where c is arbitrary constant.** **[5]**

ii) Find the order of the differential equation found in part i).

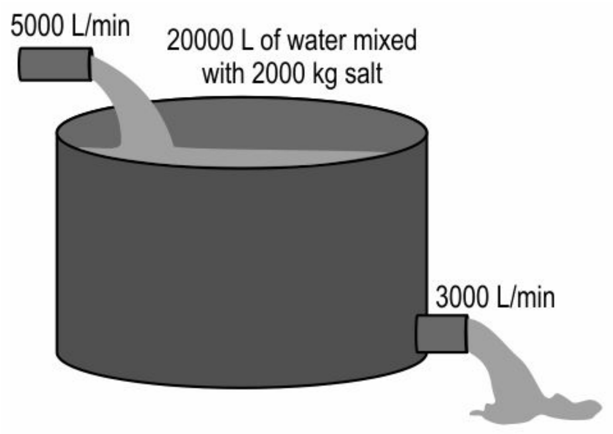
iii) Find the degree of the differential equation found in part i) if defined.

Show your work.

Case Study

Answer the questions based on the given information.

The mixing tank shown below generates saline water (a mixture of salt and water) for the cooling of a thermoelectric power plant.



The tank initially holds 20000 L of water in which 2000 kg of salt has been dissolved. Then, pure water is poured into the tank at a rate of 5000 L per minute. The mixture in the tank, which is stirred continuously, flows out at a rate of 3000 L per minute.

The quantity of salt in the tank at time t is denoted by Q_t, where t is in minutes and Q_t is in kilograms.

The rate of flow of salt into the tank is measured as:

$$\left(\frac{dQ_t}{dt}\right)_{in} = 0 \text{ kg/min}$$

The rate of flow of salt out of the tank is measured as:

$$\left(\frac{dQ_t}{dt}\right)_{out} = \text{Rate of flow of water out of the tank} \times \frac{\text{Quantity of salt in the tank at time } t}{\text{Amount of water in the tank at time } t}$$

The rate of change of quantity of salt in the tank with respect to time is given by

$$\left[\left(\frac{dQ_t}{dt}\right)_{in} - \left(\frac{dQ_t}{dt}\right)_{out}\right]$$

Q: 11 **Find the expression for the rate of change of quantity of salt in the tank with time t. Show your work.** **[1]**

Q: 12 **Find the general solution of the differential equation corresponding to the rate of change of quantity of salt in the tank with time t. Show your work.** **[2]**

Q: 13 **Use the initial conditions to determine the particular solution for the differential equation obtained. Show your work.** **[2]**

Q.No	What to look for	Marks
1	**Rewrites the given differential equation as** $ydy = e^{3x}\,dx$.	**0.5**
	Concludes that Sumit's general solution is incorrect by integrating both sides of the above equation to find the general solution as: $y^2 = \frac{2}{3} e^{3x}$ **+ C, where C is the arbitrary constant.**	**0.5**
2	**Writes a differential equation that depicts the given scenario as:** $\frac{dV}{dt} = k\sqrt[3]{\frac{dh}{dt}}$ **where,** V **is the volume,** h **is the height of the juice and** t **is the time.**	**1**
3	**Writes that the statement is false.**	**0.5**
	Gives a reason. For example, the order of the given differential equation is 4, but its degree is 2.	**0.5**
4	**Writes that the statement is false.**	**0.5**
	Gives a reason. For example, the number of arbitrary constants in the general solution of a differential equation is determined by its order, not its degree. Since the order of the given equation is 4, it will have only 4 arbitrary constants.	**0.5**
5	**Writes that** $\frac{dy}{dx} = y - \frac{2y}{x+1} = y\,(1 - \frac{2}{x+1})$. **Separates the variables as follows:** $\frac{dy}{y} = (1 - \frac{2}{x+1})\,dx$	**0.5**
	Integrates the above equation to get: $\log_e y = x - 2\log_e (x+1) + c$ $\Rightarrow \log_e y = x - \log_e (x+1)^2 + c$ $\Rightarrow \log_e y + \log_e (x+1)^2 = x + c$ **where c is a constant of integration.**	**1**

Q.No	What to look for	Marks
	Solves the above equation to find the equation of the curve as follows: $\log_e(y.(x+1)^2) = x + c$ $\Rightarrow y.(x+1)^2 = e^{x+c}$ $\therefore y = \frac{ke^x}{(x+1)^2}$, where k is a constant	**0.5**
6	**Takes V, S and *r* to be the volume, surface area and radius of the balloon at time *t*.** **Writes that since the volume of the balloon is decreasing with time, the rate of change of its radius is negative.**	**0.5**
	Uses the given information and expresses the relationship between volume and surface area as: $\frac{dV}{dt} = -k \times S$, **where *k* is a positive real number**	**0.5**
	Differentiates the above equation after substituting the expressions for volume and surface area of a sphere to get the required differential equation as: $\frac{4}{3}\pi(3r^2)\frac{dr}{dt} = -k \times 4\pi r^2$ $\Rightarrow \frac{dr}{dt} = -k$	**1**
7	**Integrates the given differential equation as:** $\int \frac{dy}{7+y} = \int \frac{dx}{2x}$ $\Rightarrow \log(7+y) = \frac{1}{2}\log x + \log c$ $\Rightarrow (7+y) = \sqrt{x}\, c$ $\Rightarrow (7+y)^2 = x\, c^2$ **where *c* is the constant of integration.**	**1.5**

Q.No	What to look for	Marks
	Compares the above equation with the general equation of parabola and concludes that the solution of the given differential equation represents the family of parabola.	**0.5**
8	**Rewrites the given differential equation as** $\frac{dy}{dx} = 3 - 2\frac{y}{x}$ **.** **Takes** $y = vx$ **and differentiates it to get** $\frac{dy}{dx} = v + x\frac{dv}{dx}$ **.**	**1**
	Rewrites the differential equation using the substitution in the above step as: $\frac{dv}{(1-v)} = 3\frac{dx}{x}$	**0.5**
	Integrates the above equation as: $-\log\|1 - v\| + \log\|C_1\| = 3\log\|x\|$ **where** C_1 **is the arbitrary constant.** **(Award full marks for equivalent appropriate answers.)**	**0.5**
	Simplifies the equation further and substitutes v **as** $\frac{y}{x}$ **to obtain the general solution of the given differential equation as:** $\left\|\frac{C_1}{1-v}\right\| = x^3$ $\Rightarrow \pm C_1.\frac{x}{(x-y)} = x^3$ $\Rightarrow x^3 = \frac{x}{(x-y)}C, \text{ where } C = \pm C_1$ **(Award full marks for equivalent appropriate answers.)**	**0.5**
	Concludes that: $f(x, y) = \frac{x}{(x-y)}$	**0.5**
9	**Uses the variable separable form to rewrite the given equation as:** $\frac{dy}{y} = k\,dt$	**0.5**

Q.No	What to look for	Marks
	Integrates the above equation on both sides to get: **ln y = kt + c, where c is a constant.**	**0.5**
	Writes the above equation in terms of e as: $y = e^{(kt+c)}$ **=>** $y = e^{kt} \times e^{c}$	**1**
	Uses the given conditions to write: **At t = 0, $y = e^{c} = e^{10}$, where e^{10} is the initial population.**	**1**
	Uses the given condition to write: **At t = 5,** $y = \frac{3}{4} e^{10} = e^{5k} \times e^{10}$ **=>** $e^{5k} = \frac{3}{4}$	**1**
	Takes the natural logarithm on both sides to find the value of k as -0.058. The working may look as follows: **5k = ln 3 - ln 4** **=> 5k = 1.09 - 1.38** **=> 5 k = -0.29** **=>** $k = \frac{-0.29}{5} = -0.058$	**1**
10	**i) Differentiates the given equation with respect to x as:** $2(y - c)\,y' = 3x^2$	**0.5**
	Simplifies the above differential equation to express c in terms of x, y and y' as: $c = y - \frac{3x^2}{2y'}$	**1**

Q.No	What to look for	Marks
	Uses the expression obtained in the above step to eliminate *c* from the original equation as: $[y-(y-\frac{3x^2}{2y'})]^2 = x^3$ $\Rightarrow 9x^4 = 4x^3(y')^2$ $\Rightarrow 4x^3(y')^2 - 9x^4 = 0$	**1.5**
	ii) Writes the order of the above differential equation as 1.	**1**
	iii) Writes the degree of the above differential equation as 2. **(If the differential equation obtained by the student is incorrect but have identified the order and degree of the incorrect equation correctly, then award 2 marks.)**	**1**
11	**Uses the information given to find the rate of change of quantity of salt as follows:** $\frac{dQ_t}{dt} = 0 - 3000 \times \frac{Q_t}{20000 + 5000t - 3000t} = \frac{-3Q_t}{20 + 2t}$	**1**
12	**Separates the variables of the differential equation as follows:** $\frac{dQ_t}{dt} = \frac{-3Q_t}{20 + 2t}$ $\Rightarrow \frac{dQ_t}{-3Q_t} = \frac{dt}{20 + 2t}$	**1**
	Integrates both sides to obtain the following: $\int \frac{dQ_t}{-3Q_t} = \int \frac{dt}{20 + 2t}$ $\Rightarrow -\frac{1}{3} \ln Q_t = \frac{1}{2} \ln (10 + t) + C$ **where C is an arbitrary constant.**	**1**

Q.No	What to look for	Marks
13	**Takes initial condition $Q_t(0) = 2000$ to obtain C as follows:** $-\frac{1}{3}\ln 2000 = \frac{1}{2}\ln 10 + C$ $C = -\frac{1}{3}\ln 2 - \frac{3}{2}\ln 10$	1
	Frames the equation as: $-\frac{1}{3}\ln Q_t = \frac{1}{2}\ln(10+t) - \frac{1}{3}\ln 2 - \frac{3}{2}\ln 10$	1

Chapter - 10
Vector Algebra

Multiple Choice Question

Q: 1 $\vec{u}$, $\vec{v}$ and $\vec{w}$ are three non-zero vectors that are neither parallel nor perpendicular to each other.
For which of the following will the product DEFINITELY be a vector?

(i) $(\vec{u} \cdot \vec{v}) \times \vec{w}$
(ii) $(\vec{u} \times \vec{v}) \cdot \vec{w}$
(iii) $(\vec{u} \times \vec{v}) \times \vec{w}$

1 only (ii)
2 only (iii)
3 only (i) and (iii)
4 all - (i), (ii) and (iii)

Free Response Questions

Q: 2 $\vec{p}$ and $\vec{q}$ are two collinear vectors. **[1]**

State whether the statement below is true or false.
Give a valid reason for your answer.

$(\lambda\vec{p} + \beta\vec{q}) \times \vec{q} = 0$, where λ and β are scalars.

Q: 3 There are two vectors $\vec{u}$ and $\vec{v}$ such that $\vec{u} \cdot \vec{v} = 0$. **[1]**

Find the projection of the vector $\vec{u}$ on the vector $\vec{v}$.
Give a valid reason for your answer.

Q: 4 **State whether the following statement is true or false. Give a valid reason.** **[1]**

If the angle between $\vec{p}$ and $\vec{q}$ is obtuse, then $\vec{p} \cdot \vec{q} < 0$.

Q: 5 If $\vec{p}$ and $\vec{q}$ are unit vectors, show that the angle between $\vec{p}$ and $\vec{p} + \vec{q}$ is the same as the angle between $\vec{q}$ and $\vec{p} + \vec{q}$. **[1]**

Q: 6 The adjacent sides of a parallelogram, PQRS, are represented by $\vec{a}$ and $\vec{b}$. **[2]**

If $\vec{a} \cdot \vec{b} = 0$, what type of parallelogram is PQRS? Give a valid reason.

Q: 7 Show that any two non-zero vectors $\vec{u}$ and $\vec{v}$ are perpendicular if $\left|\vec{u} - \vec{v}\right| = \left|\vec{u} + \vec{v}\right|$. **[5]**

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	3

Q.No	What to look for	Marks
2	Writes true.	0.5
	Gives a valid reason. For example, since $\vec{p}$ and $\vec{q}$ are collinear, $(\lambda\vec{p} + \beta\vec{q})$ is collinear to $\vec{p}$. This means that $(\lambda\vec{p} + \beta\vec{q})$ is parallel to $\vec{p}$. Hence, their cross product is zero.	0.5
3	**Writes that the projection vector will be a zero vector.**	0.5
	Gives the reason that the projection of vector $\vec{u}$ on vector $\vec{v}$ is given by $\frac{\vec{u}\cdot\vec{v}}{\lvert\vec{v}\rvert}$ and since the dot product is 0, the projection vector is a zero vector.	0.5
4	**Writes true.**	0.5
	Justifies as follows: $\vec{p} \cdot \vec{q} = p \times q \times \cos\theta$, where $\cos\theta$ is negative when $90^\circ < \theta < 180^\circ$.	0.5
5	Writes that $\vec{p} + \vec{q}$ represents the third side of the triangle whose other two sides are $\vec{p}$ and $\vec{q}$ with magnitude 1 unit each(as they are unit vectors). (Award full marks if pictorial/vector explanation is provided.)	0.5
	Concludes that both $\vec{p}$ and $\vec{q}$ make equal angles with $\vec{p} + \vec{q}$ as they form an isosceles triangle.	0.5
6	Writes that as $\vec{a} \cdot \vec{b} = 0$, $\lvert\vec{a}\rvert \times \lvert\vec{b}\rvert \times \cos\theta = 0$.	0.5

Q.No	What to look for	Marks
	Reasons that $\lvert\vec{a}\rvert \neq 0$ and $\lvert\vec{b}\rvert \neq 0$ as they need to form a parallelogram.	0.5
	Deduces that cos $\theta = 0$ and hence $\theta = 90°$.	0.5
	Concludes that PQRS is a rectangle as the sides are intersecting at 90°.	0.5
7	Considers vectors $\vec{u}$ and $\vec{v}$ as: $\vec{u} = p\hat{i} + q\hat{j} + r\hat{k}$ $\vec{v} = x\hat{i} + y\hat{j} + z\hat{k}$	0.5
	Uses the condition $\lvert\vec{u} - \vec{v}\rvert = \lvert\vec{u} + \vec{v}\rvert$ to write the following: $(p - x)^2 + (q - y)^2 + (r - z)^2 = (p + x)^2 + (q + y)^2 + (r + z)^2$	1.5
	Simplifies the above equation to obtain $px + qy + rz = 0$.	1.5
	Uses the above step to find the dot product of the two vectors $\vec{u}$ and $\vec{v}$ as: $\vec{u} \cdot \vec{v} = px + qy + rz = 0$	1
	Writes that the vectors are perpendicular since their dot product is equal to zero.	0.5

Chapter - 11
Three Dimensional Geometry

Multiple Choice Questions

Q: 1 **Given below are the vector equations of two planes.**

$$\left[\vec{r} - \left(4\hat{i} + 3\hat{j} - 4\hat{k}\right)\right] \cdot \left(\hat{i} + 4\hat{j} - \hat{k}\right) = 0$$

$$\left[\vec{r} - \left(6\hat{i} + \hat{j} - 5\hat{k}\right)\right] \cdot \left(\hat{i} + 4\hat{j} - \hat{k}\right) = 0$$

Which of the following is true about these planes?

1. They coincide with each other.
2. They are parallel to each other.
3. They are perpendicular to each other.
4. They are not perpendicular but intersect each other along a unique line.

Q: 2 **Shown below are equations of four planes.**

P_1 : $-2x - 3y - 4z - 20 = 0$
P_2 : $2x + 3y + 4z - 9 = 0$
P_3 : $-4x - 6y + 8z = 0$
P_4 : $4x + 6y + 8z = 0$

Which of the provided planes does NOT share parallelism with the rest?

1. P_1
2. P_2
3. P_3
4. P_4

Free Response Questions

Q: 3 **There are three points P, Q and R. The direction ratios of line PQ are (-2), 1 and 3 and that of line QR are 6, (-3) and (-9).** **[1]**

Is there a unique plane that contains the points P, Q and R? Justify your answer.

Q: 4 **The equations for line *m* and line *n* are given below.** **[1]**

Line *m*: $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{2}$

Line *n*: $\frac{x-1}{2} = \frac{y-1}{1} = \frac{1-z}{k}$

If line *m* is perpendicular to line *n*, find the value of *k*. Show your steps.

(Note: $k \neq 0$.)

Q: 5 **Line *m* and line *n* are two lines which are not parallel, and do not intersect each other.** **[1]**

Asha, Ravi and Saquib tried to find the shortest distance between lines *m* and *n*.

♦ Asha found a line *k* parallel to line *m* on the same plane as line *n*. She found the shortest distance as the distance between lines *k* and *n*.

♦ Ravi found the length of a line segment that is perpendicular to both line *m* and line *n* as the shortest distance.

♦ Saquib said that it is impossible to find the distance between the two lines, since the lines are not parallel, and do not intersect.

Which of them is/are using the correct approach? Justify your answer.

Q: 6 **[1]**

$\vec{b}$ is a vector parallel to a line $\vec{c}$.

$\vec{n}$ is the normal from the origin to the plane in which $\vec{c}$ lies.

Find $\vec{b} \cdot \vec{n}$. Give a valid reason.

Q: 7 **Shown below are the equations of two planes.** **[1]**

$P_1 : 3x - 3y - 3z = 1$
$P_2 : -3x + ny - z = 4$

If P_1 is perpendicular to P_2, find the value of *n*. Show your work.

Q: 8 **A line given by $\frac{x+1}{3} = \frac{y+1}{2} = \frac{2-z}{2}$ is parallel to the plane $6x + 4y + Gz = 12$.** **[2]**

Find the value of G. Show your steps.

Q: 9 **The equation of a plane can be given by $px - 2y + z = q$, where p and q ∈ R, such that:** **[2]**

♦ the plane is parallel to $-6x + 4y - 2z = 12$.

♦ the point (2, -3, 5) lies on the plane.

Find p and q. Show your work.

Q: 10 Shown below is the equation of a plane P. **[2]**

$\vec{r}.(0\hat{i} - 5\hat{j} + 0\hat{k}) = 8$

WITHOUT using the formula for distance of a point from a plane, find out how far the plane P is from the origin.

Q: 11 **Two lines are given below in vector form:** **[3]**

$$\vec{r_1} = (\hat{i} - \hat{j}) + \lambda(-\hat{i} + \hat{j} - \hat{k})$$

$$\vec{r_2} = (4\hat{i} - 3\hat{j} + \hat{k}) + \mu(-\hat{i} + 2\hat{j} - 3\hat{k})$$

Prove that the lines are non-skew lines. Give valid reasons.

Q: 12 **Shown below are the equations of a line and a plane.** **[3]**

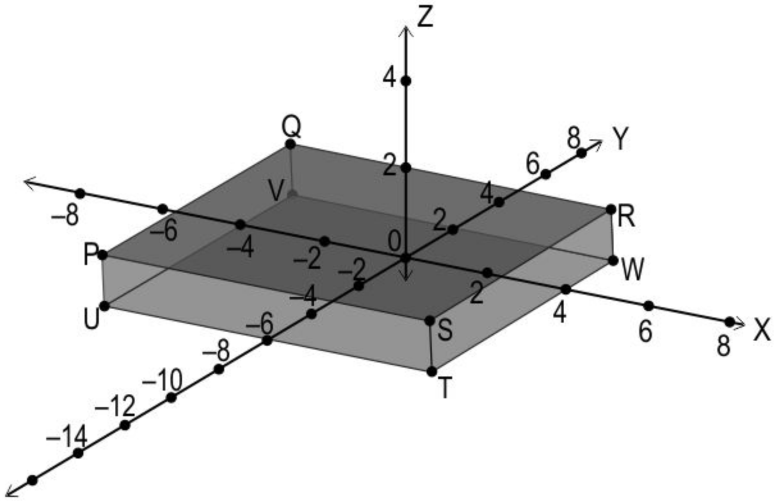
Line: $\vec{r} = \hat{i} + 2\hat{j} + \lambda\left(\hat{i} - 3\hat{j} + 2\hat{k}\right)$

Plane: $8x - 2y + 6z = 1$

where λ is some real number.

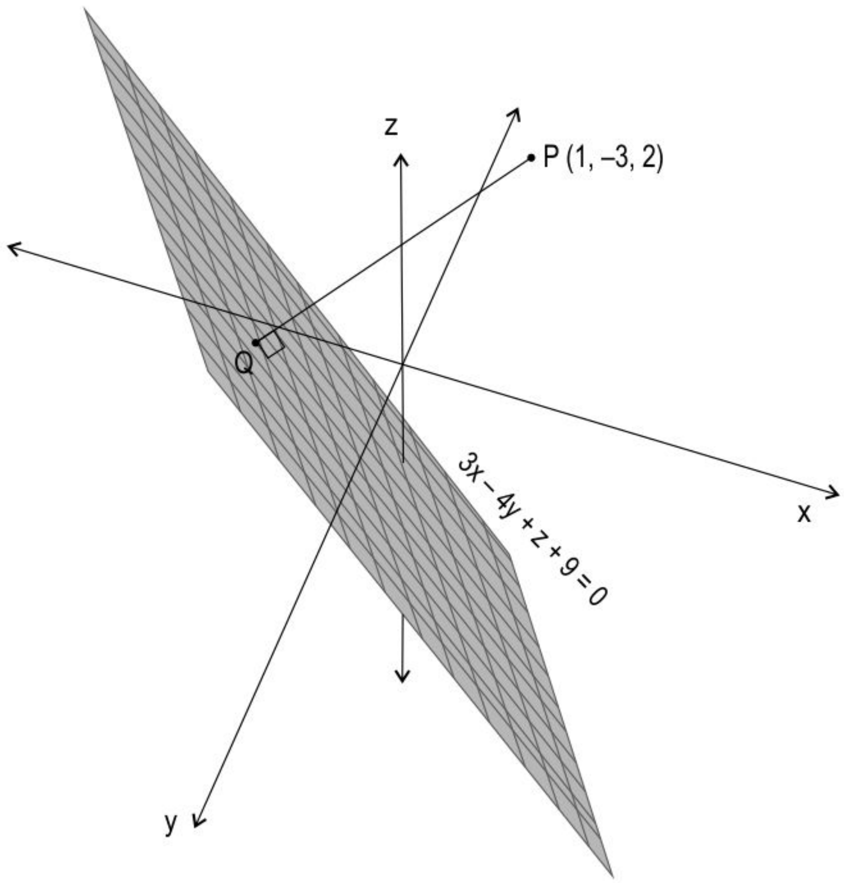
Check whether they intersect. If yes, find the point of intersection. If not, state a valid reason. Show your work.

Q: 13 **A cuboid is shown below, such that face TUVW lies on the *x* - *y* plane. The height of the cuboid is 2 units.** **[3]**



Find the equation of the line joining P and W. Does it pass through the origin? Show your work and give valid reason(s).

Q: 14 **Point P meets the plane shown below at point Q.** **[5]**



Find the coordinates of Q and the distance PQ. Show your work.

Q: 15 **The equation of line *m* is given by $\frac{x+8}{2} = \frac{5-y}{3} = \frac{z-4}{5}$. A line segment PQ is to be drawn perpendicular to line *m*, such that line *m* bisects PQ.** **[5]**

If P's coordinates are given by (-2, -7, 2), then find the coordinates of point Q. Show your steps.

Q: 16 **Shown below is an equation of a plane, where P is a constant.** **[5]**

$$3y + 4z + P = 0$$

The distance from the origin to the plane is the same as the distance from the point (n, 2, -4) to the plane. The distance from the point (2, m, 4) to the plane is six times the distance from the origin to the plane.

Find all possible values of:
i) P
ii) m
iii) n

Show your work. Give a valid reason.

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	2
2	3

Q.No	What to look for	Marks
3	**Finds that the direction ratios of PQ and QR are proportional. The working may look as follows:** $\frac{6}{-2} = \frac{-3}{1} = \frac{-9}{3}$ **= (-3), where (-3) is the constant of proportionality.**	0.5
	States that there is no unique plane because P, Q and R are collinear and infinitely many planes that contain three collinear points exist.	0.5
4	**Since the two lines are perpendicular, writes the equation:** $3(2) + 2(1) + 2(-k) = 0$	0.5
	Solves the above equation to find the value of k **as 4.**	0.5
5	**Writes that only Ravi is using the correct approach. Gives a reason.** **For example, lines** m **and** n **are skew lines, and the distance between skew lines is given by the length of a line segment that is perpendicular to both lines. Hence, Saquib is incorrect.** **Asha is incorrect as the shortest distance between two non-parallel lines on the same plane is 0.**	1
6	**States that the normal to a plane is perpendicular to every line on the plane and the lines parallel to the lines on the plane.**	0.5
	Finds the value of the dot product as: $\vec{b} \cdot \vec{n} = 0$	0.5
7	**Writes that since** P_1 **is perpendicular to** P_2**, their normals are also perpendicular. Thus equates the dot product of the normals to zero and finds the value of** n **as (-2). The working may look as follows.** $3(-3) + (-3)n + (-3)(-1) = 0$ $\Rightarrow -9 - 3n + 3 = 0$ $\Rightarrow n = -2$	1

Q.No	What to look for	Marks
8	**Writes the equation of the line in vector form as follows:** $\vec{r} = \vec{a} + \lambda\vec{b}$, where $\vec{a} = (-\hat{i} - \hat{j} + 2\hat{k})$ and $\vec{b} = (3\hat{i} + 2\hat{j} - 2\hat{k})$	0.5
	Finds the equation of the plane in vector form as $\vec{r}.\hat{n} = d$, where $d = 12$, $\hat{n} = \frac{\vec{n}}{\lvert\vec{n}\rvert}$, and $\vec{n} = (6\hat{i} + 4\hat{j} + G\hat{k})$. Hence finds the equation of the plane as $\frac{\vec{r}.(6\hat{i}+4\hat{j}+G\hat{k})}{\lvert\sqrt{6^2+4^2+G^2}\rvert} = 12$	0.5
	As the line is parallel to the plane, notes that $\sin\phi = 0$, where ϕ is the angle between the given line and plane. $\therefore\ \vec{b}.\hat{n} = 0$. Hence finds $\vec{b}.\hat{n} = (3\hat{i} + 2\hat{j} - 2\hat{k}).\frac{(6\hat{i}+4\hat{j}+G\hat{k})}{\lvert\sqrt{6^2+4^2+G^2}\rvert} = 0$ $\Rightarrow (3\hat{i} + 2\hat{j} - 2\hat{k}).(6\hat{i} + 4\hat{j} + G\hat{k}) = 0$ $\Rightarrow 18 + 8 - 2G = 0$	0.5
	Solves the above equation to find G = 13.	0.5
9	**Finds p = 3 by writing the following as the planes are parallel:** $\frac{p}{-6} = \frac{-2}{4} = \frac{1}{-2}$	1
	Finds q = 17 by substituting the coordinates of the point (2, -3, 5) in the equation of the plane as: **3(2) - 2(-3) + (5) = q**	1

Q.No	What to look for	Marks
10	**Finds the magnitude of the normal vector to the plane from the origin as:** $\left\|\overrightarrow{n}\right\| = \sqrt{0^2 + 5^2 + 0^2} = 5$	0.5
	Rewrites the given equation as: $\overrightarrow{r} . \frac{(0\hat{i} - 5\hat{j} + 0\hat{k})}{\|\overrightarrow{n}\|} = \frac{8}{\|\overrightarrow{n}\|}$ $\Rightarrow \overrightarrow{r} . \hat{n} = \frac{8}{5}$	1
	Compares the above with the equation of a plane in normal form and finds the distance between the plane and the origin as $\frac{8}{5}$ units.	0.5
11	**Considers the lines in vector form as:** $\overrightarrow{r_1} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ $\overrightarrow{r_2} = \overrightarrow{a_2} + \lambda\overrightarrow{b_2}$ which denotes that $\overrightarrow{r_1}$ passes through the point A, with position vector $\overrightarrow{a_1}$, and is parallel to $\overrightarrow{b_1}$, and $\overrightarrow{r_2}$ passes through the point B, with position vector $\overrightarrow{a_2}$, and is parallel to $\overrightarrow{b_2}$.	0.5
	Writes the vector of the lines joining A and B as: $\overrightarrow{AB} = (1 - 4)\hat{i} + (-1 - (-3))\hat{j} + (0 - 1)\hat{k} = -3\hat{i} + 2\hat{j} - 1\hat{k}$	0.5
	Writes that if the lines are coplanar, they will definitely be non-skew lines.	0.5

Q.No	What to look for	Marks
	Hence, proves coplanarity as follows: $\overrightarrow{AB}.(\overrightarrow{b_1} \times \overrightarrow{b_2}) = 0$ **the LHS of which translates to:** $\begin{vmatrix} -3 & 2 & -1 \\ -1 & 1 & -1 \\ -1 & 2 & -3 \end{vmatrix}$	**0.5**
	Finds the value of the determinant as 0.	**0.5**
	Hence, concludes that the lines are coplanar and therefore non-skew lines.	**0.5**
12	**Rearranges the equation of the line as:** $\overrightarrow{r} = (1 + \lambda)\hat{i} + (2 - 3\lambda)\hat{j} + 2\lambda\,\hat{k}$ **and finds a point P ((1 + λ), (2 - 3λ), 2λ) on the line.**	**0.5**
	Substitutes the coordinates of P in the equation of the plane as: **8(1 + λ) - 2(2 - 3λ) + 6(2λ) = 1**	**0.5**
	Finds the value of λ as (- $\frac{3}{26}$) by simplifying the above equation as follows: **8 +8λ - 4 + 6λ + 12λ = 1** **=> 4 + 26λ = 1** **=> λ = - $\frac{3}{26}$** **Writes that there exists a value of λ for which the point P satisfies the equations of both the line and the plane. Hence concludes that the line and the plane intersect.**	**1**

Q.No	What to look for	Marks
	Substitutes the value of λ in point P to find the point of intersection as: P = ($\frac{23}{26}$, $\frac{61}{26}$, - $\frac{3}{13}$)	1
13	Finds the coordinates of P as (-4, -6, 2). Finds the coordinates of W as (4, 2, 0).	1
	Finds the direction ratios of PW as follows: l = 4 - (-4) = 8 m = 2 - (-6) = 8 n = 0 - 2 = (-2)	0.5
	Assumes that (x, y, z) is an arbitrary point on the line joining P and W. Finds the equation of the line as: $\frac{x+4}{8} = \frac{y+6}{8} = \frac{2-z}{2}$ or $\frac{x-4}{8} = \frac{y-2}{8} = \frac{-z}{2}$	1
	Writes that the line does not pass through the origin, and gives a valid reason. For example, in the equation from step 3, $\frac{0+4}{8} \neq \frac{0+6}{8} \neq \frac{2-0}{2}$	0.5
14	Identifies the direction ratios of the perpendicular to the plane as 3, -4 and 1.	0.5
	Takes Q(x_1, y_1, z_1), identifies that the direction ratios of the perpendicular and the direction ratios of PQ are proportional and writes the following: $\frac{x_1-1}{3} = \frac{y_1+3}{-4} = \frac{z_1-2}{1} = k$ $\Rightarrow x_1 = 3k + 1;\ y_1 = -4k - 3;\ z_1 = k + 2$	1

Q.No	What to look for	Marks
	Substitutes the above values in the equation of the plane to obtain k = -1. The working may look as follows: **$3(3k + 1) - 4(-4k - 3) + 1(k + 2) + 9 = 0$** **$\therefore k = -1$**	**1**
	Hence, finds the coordinates of Q by substitution as Q(-2, 1, 1).	**1**
	Uses the distance formula to express PQ as follows: **$PQ = \sqrt{(1+2)^2 + (-3-1)^2 + (2-1)^2} = \sqrt{3^2 + (-4)^2 + 1^2}$ units**	**1**
	Solves the above to find the distance PQ as $\sqrt{26}$ units.	**0.5**
15	**Assumes that O(x, y, z) is the point of intersection of PQ and line m.** **Writes that $\frac{x+8}{2} = \frac{5-y}{3} = \frac{z-4}{5} = \lambda$, where λ is a constant.** **Using the equations from step 1, finds x, y, and z as follows:** **$x = 2\lambda - 8$** **$y = 5 - 3\lambda$** **$z = 5\lambda + 4$**	**1**
	Notes that O(x, y, z) and P both lie on line PQ. **Finds the direction ratios of PQ as:** **$(2\lambda - 8 - (-2), 5 - 3\lambda - (-7), 5\lambda + 4 - 2)$** **$= (2\lambda - 6, 12 - 3\lambda, 5\lambda + 2)$**	**0.5**
	Since PQ is perpendicular to line m, equates the dot product of their direction ratio to 0 to find λ as 1. **$(2\lambda - 6, 12 - 3\lambda, 5\lambda + 2).(2, -3, 5) = 0$** **$\Rightarrow 2(2\lambda - 6) - 3(12 - 3\lambda) + 5(5\lambda + 2) = 0$** **$\Rightarrow 4\lambda - 12 - 36 + 9\lambda + 25\lambda + 10 = 0$** **$\Rightarrow 38\lambda = 38$** **$\Rightarrow \lambda = 1$**	**1.5**

Q.No	What to look for	Marks
	Finds the O(x, y, z), the coordinates of the foot of the perpendicular from P to line *m* as (-6, 2, 9).	**0.5**
	Assumes the coordinates of Q as (*a, b, c*). Finds Q as (-10, 11, 16) using the midpoint theorem as: **-2 + *a* = 2(-6)** **=> *a* = -10** **-7 + *b* = 2(2)** **=> *b* = 11** **2 + *c* = 2(9)** **=> *c* = 16**	**1.5**
16	**Finds the distance from the origin to the plane as follows:** $D_1 = \frac{\lvert P\rvert}{5}$	**0.5**
	Finds the distance from the point (n, 2, -4) to the plane as follows: $D_2 = \frac{\lvert 0(n)+3(2)+4(-4)+P\rvert}{5} = \frac{\lvert -10+P\rvert}{5}$	**0.5**
	i) Uses the given condition that $D_1 = D_2$ to find the value of *P* as 5. The working may look as follows: **| *P* | = |-10 + *P* |** *P* = -10 + *P* OR - *P* = 10 - *P* OR *P* = 10 - *P* OR - *P* = -10 + *P* 0 = 10 | 0 = 10 | => 2 *P* = 10 | => -2 *P* = -10 which is not possible. | which is not possible. | => *P* = 5 | => *P* = 5 **(Award full marks if two of the unique cases are shown rather than 4 cases.)**	**1.5**
	ii) Finds the distance of the point (2, *m*, 4) from the plane as follows: $D_3 = \frac{\lvert 0(2)+3(m)+4(4)+P\rvert}{5}$	**0.5**

Inner table (part i):

P = -10 + *P*	OR	- *P* = 10 - *P*	OR	*P* = 10 - *P*	OR	- *P* = -10 + *P*
0 = 10		0 = 10		=> 2 *P* = 10		=> -2 *P* = -10
which is not possible.		which is not possible.		=> *P* = 5		=> *P* = 5

Q.No	What to look for	Marks
	Uses the given condition that $D_3 = 6D_1$ to find the value of *m* as 3 or -17. The working may look as follows: $\frac{\lvert 0(2)+3(m)+4(4)+P \rvert}{5} = \frac{6\lvert P \rvert}{5}$ **=> $\lvert 3m + 16 + 5 \rvert = 30$** $3m + 16 + 5 = 30$ OR $3m + 16 + 5 = -30$ => $3m = 9$ => $3m = -51$ => $m = 3$ => $m = -17$	1
	iii) Writes that *n* can take any value as the coefficient of *x* in the given plane is 0.	1

Chapter - 12
Linear Programming

Multiple Choice Questions

Q: 1 **Shown below is a linear programming problem (LPP).**

Maximise Z = $x + y$

subject to the constraints:

$x + y \leq 1$
$-3x + y \geq 3$
$x \geq 0$
$y \geq 0$

Which of the following is true about the feasible region of the above LPP?

1. It is bounded.
2. It is unbounded.
3. There is no feasible region for the given LPP.
4. (cannot conclude anything from the given LPP)

Q: 2 **Two statements are given below - one labelled Assertion (A) and the other labelled Reason (R). Read the statements carefully and choose the option that correctly describes statements (A) and (R).**

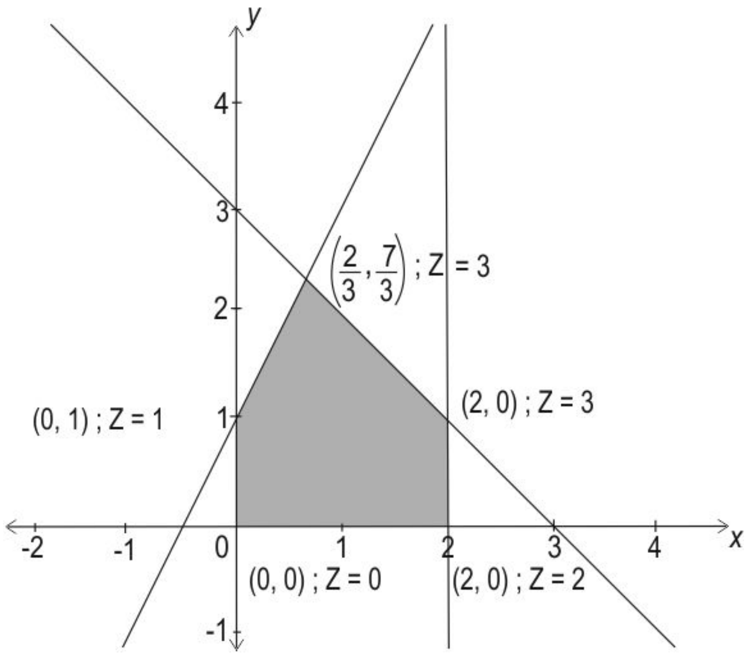
***Assertion* (*A*): All the points in the feasible region of a linear programming problem are optimal solutions to the problem.**

***Reason* (*R*): Every point in the feasible region satisfies all the constraints of a linear programming problem.**

1. Both (A) and (R) are true and (R) is the correct explanation for (A).
2. Both (A) and (R) are true but (R) is not the correct explanation for (A).
3. (A) is false but (R) is true.
4. Both (A) and (R) are false.

Free Response Questions

Q: 3 **Shown below is the feasible region of a linear programming problem(LPP) whose objective function is maximize Z = x + y.** **[1]**



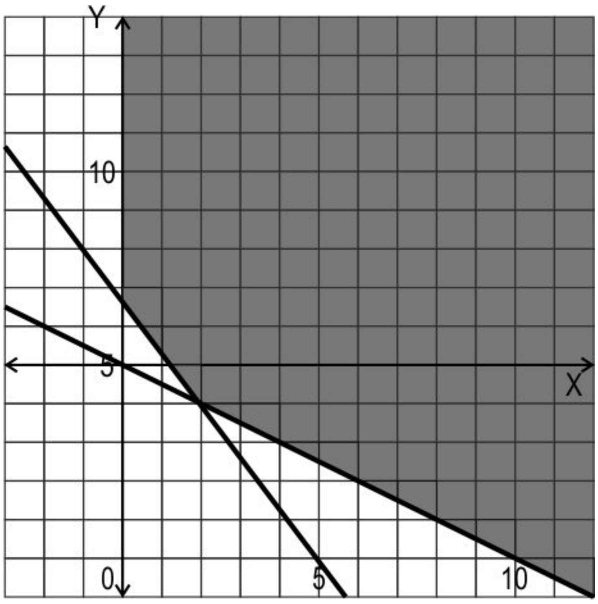
Sarla claimed that there exists no optimal solution for the LPP as there is no unique maximum value at the corner points of its feasible region.

Is her claim true? Give a valid reason.

Q: 4 **State whether the following statement is true or false. Explain your reasoning.** **[1]**

In a linear programming problem, it is possible for a non-corner point to have the optimal value of the objective function, *Z* = a *x* + b *y*.

Q: 5 **The shaded region in the graph shown below represents the feasible region for a linear programming problem with objective function Minimize: Z = 4 *x* - 3 *y*.** **[1]**

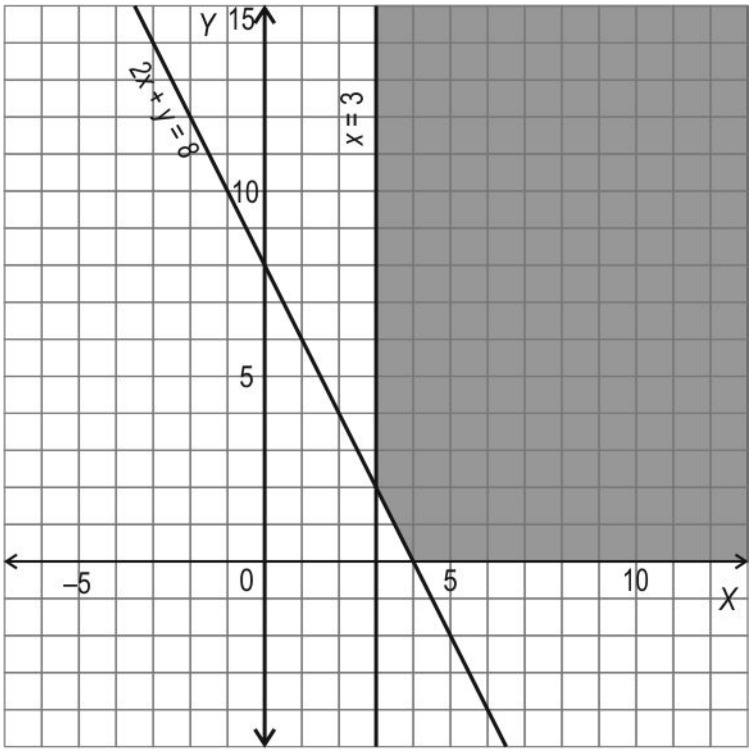


The values of the objective function at the corner points are given below:

Corner points	Z = 4x –3y
$(0, \frac{20}{3})$	–20
(2, 4)	–4
(10, 0)	40

Suhas says that the minimum value of Z is (-20). Is he correct or incorrect? Justify your answer.

Q: 6 **A feasible region with respect to certain constraints is shaded in the graph below.** **[1]**



Reece says that the objective function Z = *x* + *y* will give an optimal solution when maximized. Is he correct or incorrect? Justify your answer.

Q: 7 **The objective function of a linear programming problem is given by:** **[2]**

Maximise Z = *ax* + *by*

The following are known about the linear programming problem:

♦ The problem has an unbounded feasible region.
♦ (x_0, y_0) is a corner point of the feasible region such that $ax_0 + by_0$ has the maximum value among all the values of Z evaluated at the corner points of the feasible region.
♦ The optimal solution of this problem does not exist.

When and why does this occur?

Q: 8 Benjamin, a professional runner, follows a rigorous workout routine designed to improve his athletic performance. Running on the treadmill is a part of his daily routine. [3]

If he runs at 10 km/hr, he burns 90 cal/km. If he runs at a faster speed of 16 km/hr, the calories burnt increases to 150 cal/km. He wishes to run maximum distance in not more than an hour at only two speeds. He does not want to burn more than 1000 calories as it could be detrimental for his health.

Express the above optimisation problem as a linear programming problem.

(Note: Treadmill is a device used for exercise, consisting of a continuous moving belt on which one can walk or run.)

Q: 9 A government bond or sovereign bond is a form of bond issued by the government to raise money for public works such as parks, libraries, bridges, roads, and other infrastructure. There are multiple variants of bonds issued by Government of India (GOI) at fixed interest rates and for a fixed period. [5]

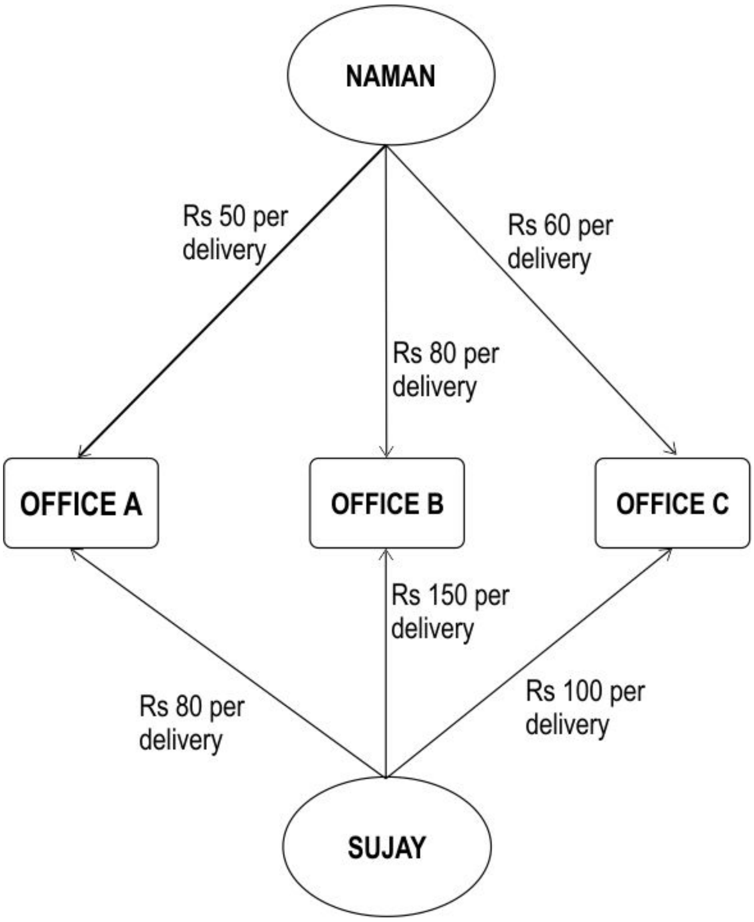
Jayesh Runjhunwala, an investor, wants to invest a maximum of Rs 20000 in government bonds for one year. He decides to invest in two types of bonds X and Y, bond X yielding 10% simple interest on the amount invested and bond Y yielding 15% simple interest on the amount invested. He wants to invest at least Rs 5000 in bond X and no more than Rs 8000 in bond Y.

Formulate the linear programming problem and determine the amount he must invest in two bonds to maximise his return. Show your work.

Q: 10 **Naman and Sujay work for Megha's tiffin service as delivery people. In their area, there are 3 offices that order lunch from Megha's tiffin service. Naman and Sujay's daily targets are to deliver 30 and 40 tiffins, respectively. They can each carry only one tiffin at a time to deliver to an office, to prevent the food from spilling out.** **[5]**

On average, the number of tiffins ordered by offices A, B and C daily are 20, 30 and 20 respectively.

The cost of travelling to/from each office to deliver a tiffin is shown in the tree diagram below:



Megha wants to minimise the cost of delivery for her tiffins.

i) Using the given tree diagram, form a system of inequalities to represent the above situation, in order to find out how many tiffins Naman and Sujay should each deliver to a respective office in a day.

ii) Find the function of the daily cost of delivery for Megha's tiffin service.

iii) Graphically solve the system of inequalities obtained in step i) to minimise the daily cost of delivery.

Show your work.

Q: 11 **Sudhanva has two kinds of areca nuts - premium and economy. He sells them to two stores, A and B. The profit margin per kg for premium and economy nuts are Rs 80 and Rs 50 respectively.** **[5]**

Both stores buy an equal quantity of premium nuts. However, Store B buys three times the economy nuts as compared to Store A. Store A can buy a maximum of 30 kg of nuts and store B can buy a maximum of 60 kg of nuts.

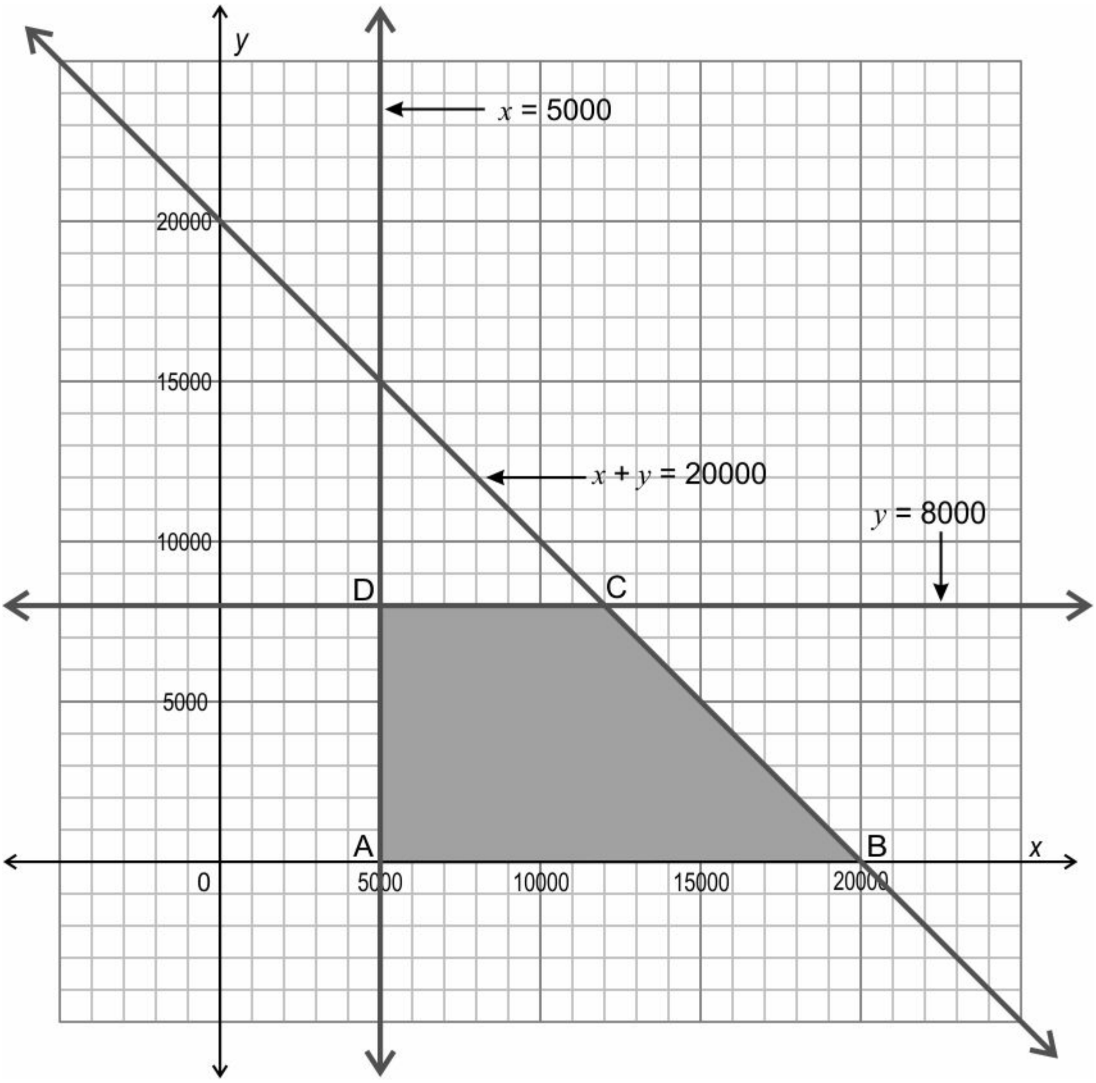
What should be the maximum weight of premium and economy nuts that Sudhanva should sell to obtain maximum profit? Show your steps.

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	3
2	3

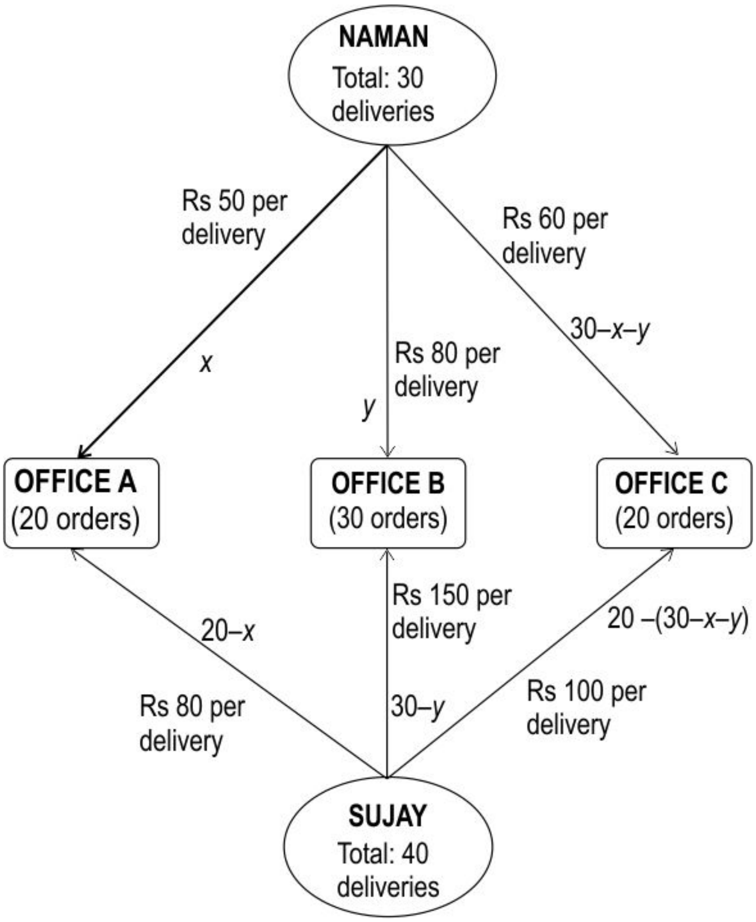
Q.No	What to look for	Marks
3	**Writes that Sarla's claim is false.**	**0.5**
	Gives a reason. For example, every point on the line joining ($\frac{2}{3}$, $\frac{7}{3}$) and (2, 1) gives the maximum value of Z which is 3. Hence, any point on the line joining ($\frac{2}{3}$, $\frac{7}{3}$) and (2, 1) is an optimal solution.	**0.5**
4	**Writes that the statement is true.**	**0.5**
	Gives a valid reason. For example, if two adjacent corner points yield the optimal value for the objective function, then every point on the line joining them also yields the optimal value.	**0.5**
5	**Writes that Suhas is incorrect.**	**0.5**
	Reasons that since the feasible area is unbounded, the minimum value of the objective function will exist only if the graph of $4x - 3y < (-20)$ has no point in common with the feasible region. Shows as an example that (1, 10) satisfies the inequality $4x - 3y < (-20)$ and is in common with the feasible region and hence, the minimum value of Z does not exist.	**0.5**
6	**Writes that he is incorrect.**	**0.5**
	Finds the value of the objective function at the corner points (2, 3) and (4, 0) as 5 and 4 respectively, and writes that $x + y > 5$ is overlapping with the feasible region and hence, no maximum value exists.	**0.5**
7	**Writes that when $Z > ax_0 + by_0$ has at least one point common with the feasible region, the optimal solution of this problem does not exist.**	**0.5**
	Gives reason that when $Z > ax_0 + by_0$ has at least one point common with the feasible region, it means that there is at least one non-corner point (x_1, y_1) in the feasible region such that $ax_1 + by_1 > ax_0 + by_0$.	**1**
	Further reasons that Z has no maximum value at (x_0, y_0) and thus the optimal solution of this problem does not exist as a maximum value, if exists, has to be obtained at a corner point .	**0.5**

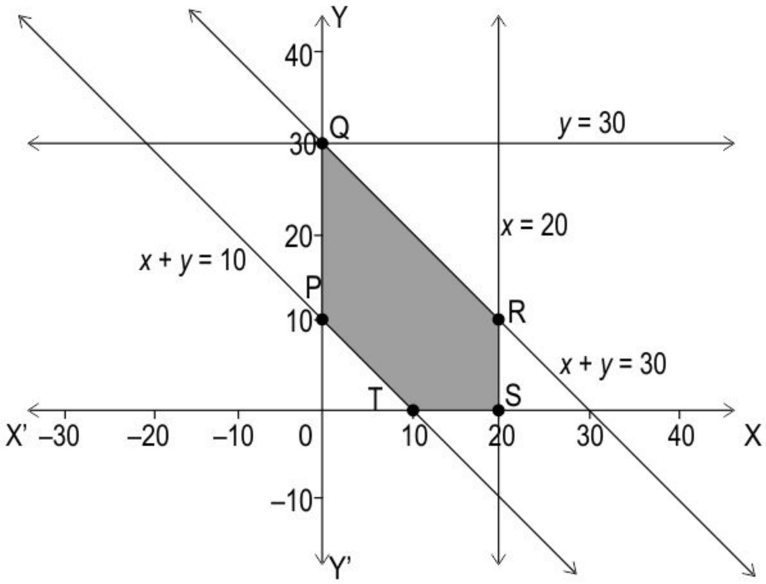
Q.No	What to look for	Marks
8	**Takes x and y to be the distances (in km) covered by Benjamin at the speeds of 10 km/hr and 16 km/hr respectively.** **Finds the time consumed in covering these distances as $\frac{x}{10}$ hr and $\frac{y}{16}$ hr respectively.**	1
	Writes the objective function of the given LPP as: **Maximise Z = $x + y$, where Z is the total distance covered.**	0.5
	Finds the calorie constraint corresponding to the given LPP as: $90x + 150y \leq 1000$	0.5
	Finds the time constraint corresponding to the given LPP as: $\frac{x}{10} + \frac{y}{16} \leq 1$	0.5
	Writes the non-negativity constraints corresponding to the given LPP as: $x \geq 0$ $y \geq 0$	0.5
9	**Assumes the amount to be invested in bonds X and Y to be Rs x and Rs y respectively.** **Frames the objective function of the given problem as:** **Maximise** $Z = \frac{x}{10} + \frac{3y}{20}$ **(Award full marks if the objective function is framed based on the total amount as Maximise** $Z = \frac{11x}{10} + \frac{23y}{20}$ **.)**	1
	Writes the constraints of the given LPP as: $x + y \leq 20000$ $x \geq 5000$ $y \leq 8000$ $x, y \geq 0$	1

Q.No	What to look for	Marks
	Draws the graph of the system of inequalities and shades the feasible region. The graph may look as follows: 	**1.5**

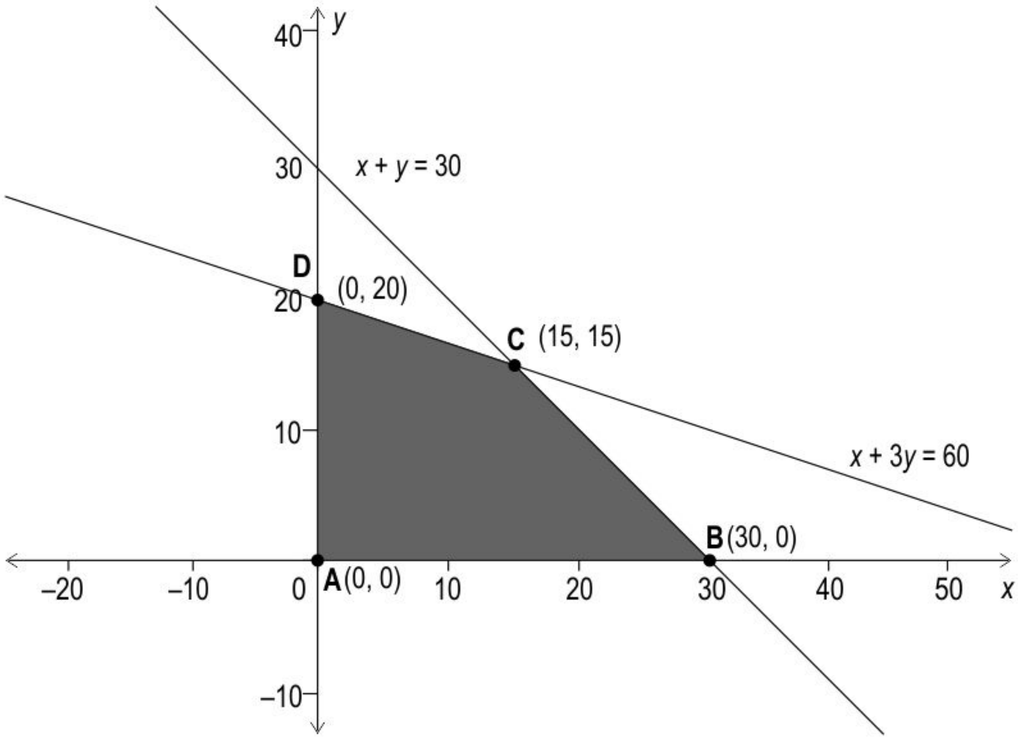
<table>
<tr><th>Q.No</th><th>What to look for</th><th>Marks</th></tr>
<tr><td></td><td>Evaluates the objective function at the corner points to find the optimal solution as follows:

<table><tr><th>Corner points</th><th>$Z = \frac{x}{10} + \frac{3y}{20}$</th></tr><tr><td>A(5000, 0)</td><td>500</td></tr><tr><td>B(20000, 0)</td><td>2000</td></tr><tr><td>C(12000, 8000)</td><td>2400</td></tr><tr><td>D(5000, 8000)</td><td>1700</td></tr></table>
(Award full marks if correct Z values are obtained corresponding to the objective function $\frac{11x}{10} + \frac{23y}{20}$.)</td><td>1</td></tr>
<tr><td></td><td>Writes that the maximum value of Z is obtained at the corner point C. Hence, concludes that (12000, 8000) is the optimal solution and Jayesh should invest Rs 12000 in bond X and Rs 8000 in bond Y.</td><td>0.5</td></tr>
</table>

Q.No	What to look for	Marks
10	**i) Considers x as the number of deliveries made by Naman to office A, and y as the number of deliveries made by Naman to office B.** **Forms the following inequalities:** $x \geq 0$, $y \geq 0$ $x + y \leq 30$ $x \leq 20$ $y \leq 30$ $x + y \geq 10$ **The completed tree diagram may look as follows:** 	1

Q.No	What to look for	Marks
	ii) Finds the delivery cost function, Z as 10 x - 30 y + 6900. **The working may look as follows:** **Delivery cost = 50 x + 80 y + 60(30 - x - y) + 80(20 - x) + 150(30 - y) + 100(x + y - 10)** **= 50 x + 80 y + 1800 - 60 x - 60 y + 1600 - 80 x + 4500 - 150 y - 1000 + 100 x + 100 y** **= 10 x - 30 y + 6900**	1
	iii) Plots lines x + y = 30, x = 20, y = 30 and x + y = 10 on a graph and finds the area enclosed by polygon PQRST as the feasible region. **The graph may look as follows:** 	1

Q.No	What to look for	Marks
	Finds the value of the cost function at each of the corner points as follows: <table><tr><th>Point</th><th>x -coordinate</th><th>y -coordinate</th><th>Cost (Z)</th></tr><tr><td>P</td><td>0</td><td>10</td><td>Rs 6600</td></tr><tr><td>Q</td><td>0</td><td>30</td><td>Rs 6000</td></tr><tr><td>R</td><td>20</td><td>10</td><td>Rs 6800</td></tr><tr><td>S</td><td>20</td><td>0</td><td>Rs 7100</td></tr><tr><td>T</td><td>10</td><td>0</td><td>Rs 7000</td></tr></table>	1
	Finds that the cost is minimum when $x = 0$ and $y = 30$. **Writes that Naman should deliver 30 tiffins to office B, and Sujay should deliver 20 tiffins each to offices A and C.**	1
11	**Assumes the weight of premium nuts for each store as x kg and the weight of economy nuts for store A as y kg. And, writes the constraints of the given problem as:** $x + y \leq 30$ $x + 3y \leq 60$ $x \geq 0$ $y \geq 0$	1.5
	Frames the objective function of the given problem as: **Maximize** $Z = (80 \times 2x) + (50 \times 4y) = 160x + 200y$	0.5

Q.No	What to look for	Marks
	Draws the graph of the system of inequalities and shades the feasible region. The graph may look as follows: 	**1.5**
	Evaluates the objective function at the corner points to find the optimal solution as follows:	**1**
	Writes that the maximum value of Z is obtained at the corner point C(15, 15) and concludes that the maximum profit of Rs 5400 is obtained when 15 kg of premium nuts are sold and 15 kg of economy nuts are sold.	**0.5**

Corner points	Z = 160 x + 200 y
A(0, 0)	0
B(30, 0)	4800
C(15, 15)	5400
D(0, 20)	4000

Chapter - 13
Probability

Free Response Questions

Q: 1 **Mumbai has a public transport system consisting of local trains which connect numerous stations across Mumbai. Among them are Andheri and Dadar.** **[1]**

Vani takes the local train scheduled at 8:55 AM from Andheri station to Dadar station every morning.

♦ **The probability that the train is late is $\frac{3}{4}$.**
♦ **The probability that Vani gets a seat in the train is $\frac{1}{15}$.**

What is the probability that the train is on-time, and Vani gets a seat in it? Show your work.

Q: 2 **A and B are events in the sample space S such that P(A) > 0 and P(B) > 0.** **[1]**

Check whether the following statement is true or false. If true, justify your conclusion; if false, state the right expression.

“If A and B are mutually exclusive, the probability of at least one of them occurring is: P(A).P(B)”

Q: 3 **A company conducts a mandatory health check up for all newly hired employees, to check for infections that could affect other office-going employees. A blood infection affects roughly 5% of the population. The probability of a false positive on the test for this infection is 4%, while the probability of a false negative on the test is 3%.** **[3]**

If a person tests positive for the infection, what is the probability that they are actually infected? Show your work.

(Note: A false positive on a test refers to a case when a person is not infected, but tests positive for the infection. A false negative on a test refers to a case when a person is infected, but tests negative for the infection.)

Q: 4 **A survey says that approximately 30% of all products ordered online across various e-commerce websites are returned. Two products that are ordered online are selected at random.** **[5]**

If X represents the number of products that are returned,

i) find the probability distribution of X.
ii) find the mean or expectation of X.

Show your work.

Q: 5 **Akshay is solving an exam paper with 50 multiple choice questions, each for 1 mark. In the paper, each multiple choice question has 4 options and exactly one correct answer.** **[5]**

Akshay solves only 30 questions, and randomly guesses the answers for the rest.

If exactly 40% of the questions Akshay solved are solved correctly, then what is the probability that Akshay scores at least 30 marks on the exam? Show your work.

Q.No	What to look for	Marks
1	**Finds the probability that the train is on-time as 1 - $\frac{3}{4} = \frac{1}{4}$.**	**0.5**
	Finds the probability that the train is on-time, and Vani gets a seat in it as $\frac{1}{4} \times \frac{1}{15} = \frac{1}{60}$.	**0.5**
2	**Writes false.**	**0.5**
	Writes the correct expression. For example, since A and B are mutually exclusive, A ∩ B = Φ, which means that P(A ∩ B) = 0. Hence, the probability that at least one of them occurs is P(A ∪ B) = P(A) + P(B). **(Award full marks if just P(A ∪ B) = P(A) + P(B) is written without any explanation.)**	**0.5**
3	**Considers:** **X = Event that employee is affected by the infection.** **Y = Event that employee tests positive for the infection.** **Writes that:** **P(Y|X') = 0.04** **P(Y'|X) = 0.03** **P(X) = 0.05**	**0.5**
	Writes that: **P(Y|X) = 1 - P(Y'|X) = 0.97**	**0.5**
	Uses the law of total probability to find P(Y) as follows: **P(Y) = P(Y|X)P(X) + P(Y|X')P(X')** **=> P(Y) = (0.97)(0.05) + (0.04)(0.95)** **=> P(Y) = 0.0485 + 0.038 = 0.0865**	**1**

Q.No	What to look for	Marks
	Uses Bayes' Theorem to find P(X|Y) as follows: $P(X\|Y) = \frac{P(Y\|X)P(X)}{P(Y)}$ $\Rightarrow P(X\|Y) = \frac{0.97 \times 0.05}{0.0865} = \frac{0.0485}{0.0865}$ $\Rightarrow P(X\|Y) = \frac{97}{173}$	1
4	**i) Writes that the possible values of X are 0, 1 and 2.**	0.5
	Takes R to be the event that the product is returned and N to be the event that the product is not returned. **Uses the given information and finds the probability that the product is returned as P(R) = 0.3 and the probability that the product is not returned as P(N) = 1 - 0.3 = 0.7.**	1
	Finds the probability of each possible values of the random variable X as: **P(X = 0) = P(NN) = (0.7)(0.7) = 0.49** **P(X = 1) = P(RN, NR) = (0.3)(0.7) + (0.7)(0.3) = 0.21 + 0.21 = 0.42** **P(X = 2) = P(RR) = (0.3)(0.3) = 0.09**	2
	Finds the probability distribution of X as: X or x_i: 0, 1, 2 P or p_i: 0.49, 0.42, 0.09	0.5
	ii) Finds the mean or expectation of X as: **E(X) = $\sum x_i p_i$ = (0 × 0.49) + (1 × 0.42) + (2 × 0.09) = 0.6**	1

X or x_i	0	1	2
P or p_i	0.49	0.42	0.09

Q.No	What to look for	Marks
5	**Finds the number of questions Akshay solved correctly as 40% of 30 = 12.**	**1**
	Finds the number of questions Akshay randomly guessed the answer for as 20. **∴ The number of trials, $n = 20$** **Assumes p as the probability that the answer is correct, and q as the probability that the answer is incorrect.** **Finds $p = \frac{1}{4}$ and $q = \frac{3}{4}$.**	**1.5**
	Finds that the required probability, P(at least 30 marks) = P(at least 18 correct guesses) = P(exactly 18 correct guesses) + P(exactly 19 correct guesses) + P(exactly 20 correct guesses).	**0.5**
	Evaluates this using the Binomial Distribution as follows: P(exactly 18 correct guesses) = $^{20}C_{18}\left(\frac{1}{4}\right)^{18}\left(\frac{3}{4}\right)^{2}$ $= \frac{20!}{18!\times 2!} \times \frac{9}{4^{20}} = \frac{1710}{4^{20}}$ P(exactly 19 correct guesses) = $^{20}C_{19}\left(\frac{1}{4}\right)^{19}\left(\frac{3}{4}\right)^{1}$ $= \frac{20!}{19!\times 1!} \times \frac{3}{4^{20}} = \frac{60}{4^{20}}$ P(exactly 20 correct guesses) = $^{20}C_{20}\left(\frac{1}{4}\right)^{20}\left(\frac{3}{4}\right)^{0} = \frac{1}{4^{20}}$	**1.5**
	Finds the required probability as: $\frac{1710}{4^{20}} + \frac{60}{4^{20}} + \frac{1}{4^{20}} = \frac{1771}{4^{20}}$	**0.5**

14. Annexure

Correct Answer Explanation

Chapter Name	Q.No	Correct Answer	Correct Answer Explanation
Determinants	1	1	The "assertion" is a special case of "reasoning" where every row of V is multiplied by the scalar 'k'. Thus both assertion and the reasoning are true and latter is the correct explanation for former. Hence, option A is the correct answer.
Continuity and Differentiability	2	4	The product of the two functions is given by f.g = |x|. |x| is continuous everywhere in R. Hence, option D is the correct answer.
Application of Integrals	1	3	The shaded region is the difference of the areas covered by g(y) and f(y) with the y-axis. The limit of integration is from "c" to "d" since the integration is with respect to y-axis. Hence, option C is the correct answer.
Three Dimensional Geometry	2	3	In P1 and P2, (-2/2) = (-3/3) = (-4/4) = -1. In P1 and P3, (-2/-4) = (-3/-6) ≠ (-4/8). In P1 and P4, (-2/4) = (-3/6) = (-4/8). So, P1, P2 and P4 are parallel to each other and P3 does not share parallism with the rest. Hence, option B is the correct answer.

Central Board of Secondary Education
Shiksha Sadan, 17, Rouse Avenue,
New Delhi-110002